Problem 1 (10%). The figure below shows a set of 5 segments. Give the trapezoidal map that is decided by these segments.

Problem 2 (10%). The left figure below shows the Delaunay triangulation of the set of black points. Suppose that we want to insert point \( p \) (i.e., the white point). Draw the resulting Delaunay triangulation in the figure on the right.

Problem 3 (20%). Let \( P \) be a set of \( n \) points in \( \mathbb{R}^2 \). Given an axis-parallel rectangle \( q \), a query reports the number of points in \( q \cap P \). Describe a data structure of \( O(n) \) size that answers such a query in \( O(\sqrt{n}) \) time.

Problem 4 (20%). Let \( S \) be a set of horizontal segments in \( \mathbb{R}^2 \), where each segment has the form \([x_1, x_2] \times y\). Given a point \( q \), a query reports the first segment of \( S \) that will be hit if we shoot a ray
upwards from \( q \) (e.g., in the figure below, the query reports \( s \)). Preprocess \( S \) into a data structure of \( O(n) \) space such that a query can be answered in \( O(\log n) \) time.

![Diagram](image)

**Problem 5 (20%).** Let \( S \) and \( T \) be two sets of points in \( \mathbb{R}^2 \). Let \((p,q)\) be a closest pair of \( S \) and \( T \), namely, the Euclidean distance between \( p \) and \( q \) is the smallest among all pairs of points in \( S \times T \). For example, in the figure below, let \( S (T) \) be the set of black (white) points. The closest pair is the two points between which there is a segment. Prove that there must be an edge between \( p \) and \( q \) in the Delaunay triangulation of \( S \cup T \).

![Diagram](image)

**Problem 6 (20%).** Let \( P \) be a set of \( n \) points in \( \mathbb{R}^2 \). Given a rectangle \( r \) and a query point \( q \), a constrained nearest neighbor query returns the point in \( P \cap r \) that has the smallest Euclidean distance to \( q \) (i.e., among all the points of \( P \) falling in \( r \), report the one closest to \( q \)). For example, in the figure below, let \( P \) be the set of black points; given the rectangle \( r \) and \( q \) as shown, a query returns point \( p_1 \) as its answer (note that the answer is not \( p_2 \) as it is outside \( r \)). Give a structure of \( O(n \log^2 n) \) space that answers such a query in \( O(\log^3 n) \) time.

![Diagram](image)