CSCI5010 Exercise List 9

Problem 1 (Segment-Ray Intersection). Let \( S \) be a set of horizontal rays in \( \mathbb{R}^2 \), where each ray has the form \([x, \infty) \times y\). Given a vertical segment \( q = x \times [y_1, y_2] \), a query reports all the rays in \( S \) intersecting \( q \) (e.g., in the figure below, the query reports two rays). Preprocess \( S \) into a data structure of \( O(n) \) space such that a query can be answered in \( O(\log n + k) \) time, where \( k \) is the number of rays reported.

Problem 2 (Segment-Segment Intersection). Let \( S \) be a set of horizontal segments in \( \mathbb{R}^2 \), where each segment has the form \([x_1, x_2] \times y\). Given a vertical segment \( q = x \times [y_1, y_2] \), a query reports all the segments in \( S \) intersecting \( q \) (e.g., in the figure below, the query reports two segments). Preprocess \( S \) into a data structure of \( O(n) \) space such that a query can be answered in \( O(\log^2 n + k) \) time, where \( k \) is the number of segments reported. (Hint: combine an interval tree and priority search trees.)

Problem 3 (Ray Shooting). Let \( S \) be a set of horizontal segments in \( \mathbb{R}^2 \), where each segment has the form \([x_1, x_2] \times y\). Given a point \( q \), a query reports the first segment of \( S \) that will be hit if we shoot a ray upwards from \( q \) (e.g., in the figure below, the query reports \( s \)). Preprocess \( S \) into a data structure of \( O(n) \) space such that a query can be answered in \( O(\log n) \) time. (Hint: convert the problem into a point location problem.)