A CSP is a triple $\langle Z, D, C \rangle$

- $Z$ - a finite set of variables $\{x_1, x_2, \ldots, x_n\}$
- $D$ - a function that maps each variable $x$ to its domain $D(x)$, a (possibly infinite) set of objects
- $C$ - a constraint, each primitive constraint of which on an arbitrary subsets of $Z$
Rabbits and Pheasants

- There are rabbits and pheasants in the a large field. We are told that there are 20 head and 56 legs all total. How many rabbits and pheasants are there?

- What are the variables, domains and constraints in this problem?
Rabbits and Pheasants

- Variables
  - rabbits, pheasants

- Domains
  - \( D(\text{rabbits}) = D(\text{pheasants}) = \{0,1,\ldots,20\} \)

- Constraints
  - \( \text{rabbits} + \text{pheasants} = 20 \)
  - \( (4 \times \text{rabbits}) + (2 \times \text{pheasants}) = 56 \)
Cryptarithmetic Puzzles

\[
\begin{align*}
S & E & N & D \\
+ ) & M & O & R & E \\
\hline
M & O & N & E & Y
\end{align*}
\]

\[
\begin{align*}
D & O & N & A & L & D \\
+ ) & G & E & R & A & L & D \\
\hline
R & O & B & E & R & T
\end{align*}
\]
Send + More = Money

- **Variables**
  - S, E, N, D, M, O, R, Y

- **Domains**
  - \(D(S) = D(E) = D(N) = D(D) = \{0, 1, \ldots, 9\}\)
  - \(D(M) = D(O) = D(R) = D(Y) = \{0, 1, \ldots, 9\}\)

- **Constraints**
  - \((1000xS + 100xE + 10xN + D) + (1000xM + 100xO + 10xR + E) = (10000xM + 1000xO + 100xN + 10xE + Y)\)
  - All variables take different value
N-Queens Problem

- To place $n$ Queens to a $n \times n$ chess board such that none of them can capture any others.
N-Queens Problem

- Variables
  - $X_i, Y_i$ where $i, j = 1, \ldots, n$

- Domains
  - $D(X_i) = D(Y_i) = \{1, \ldots, n\}$

- Constraints
  - $X_i \neq X_j$ if $i \neq j$, $i, j = 1, \ldots, n$
  - $Y_i \neq Y_j$ if $i \neq j$, $i, j = 1, \ldots, n$
  - $|X_i - X_j| \neq |Y_i - Y_j|$ if $i \neq j$, $i, j = 1, \ldots, n$
Scene Labelling Problem

- Probably the first constraint satisfaction problem
- Recognize objects in 3D scene by interpreting lines in 2D drawings
Concave & Convex

By M. C. Escher
An Impossible Polyhedra
David L. Waltz initially proposed the filtering algorithm as a way to reduce combinatorics associated with line labeling of 3D scenes.

This algorithm popularized the technique of constraint propagation and is known as the Waltz Filtering Algorithm [Waltz 1972].
Four Kinds of Valid Junctions

- L Junction
- Fork Junction
- T Junction
- Arrow Junction
Labeling an Object

Convex edge

Boundary edge

Concave edge
Four Correct Labelings of a Cube
CSP Modeling

- Modeling the scene labeling problem as a CSP
- What are the variables, domains and constraints in this problem?

Hints: represent each unknown edge label as a constrained variable
Any Questions...