Patricia Tries

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We will continue the discussion of the exact matching problem on strings.

Problem

Let S be a set of strings, each of which has a unique integer id. Given a query string q, a query reports:

- the id of q if it exists in S
- nothing otherwise.

Notations

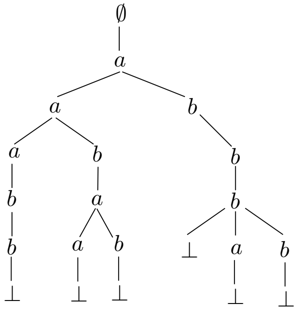
Let

- A be the alphabet (i.e., every character of any string must come from A).
- $|s|$ be the length of a string s, i.e., the number of characters in s.
- $m = |S|$, i.e., the number of strings in S.
- $n =$ the total length of the strings in S, i.e., $n = \sum_{s \in S} |s|$.

So far, all our tries use $O(n)$ space. In this lecture, we will improve the space consumption to $O(m)$, without affecting the query time.

This is achieved by a variant of tries called the Patricia trie.

Let $S = \{\texttt{aaabb}\bot, \texttt{aabaa}\bot, \texttt{aabab}\bot, \texttt{abbb}\bot, \texttt{abbba}\bot, \texttt{abbbb}\bot\}$. The trie of S is:



A trie can have many internal nodes that have only one child. A Patricia trie essentially eliminates all such nodes.

We will from now on denote the strings in S as $s_1, s_2, ..., s_m$, respectively. We will consider that each s_i is stored in an array of size $|s_i|$, such that $s_i[j]$ gives the j-th $(1 \leq j \leq |s_i|)$ character of s_i.

Definition (Longest Common Prefix)

The longest common prefix (LCS) of a set S of strings is a string σ such that:

- σ is a prefix of every string in S.
- There is no string σ' such that σ' is a prefix of every string in S, and $|\sigma'| > |\sigma|$.

For example, the LCS of $\{\texttt{aaabb}\bot, \texttt{aab}\bot, \texttt{aabaa}\bot\}$ is aa, and the of LCS of $\{\texttt{aaabb}\bot, \texttt{baa}\bot\}$ is $\emptyset$.

Given two strings s_1, s_2, we use $s_1 \cdot s_2$ to denote their concatenation.

Definition (Extension Set)

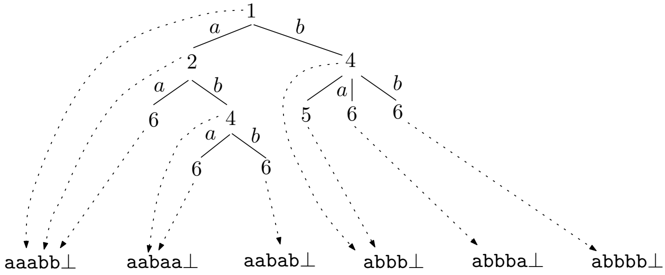
Let S be a set of strings, and σ the LCS of S. The extension set of S is the set of characters c such that $\sigma \cdot c$ is a prefix of at least one string in S.

For example, the extension set of $\{\texttt{aaabb}\perp, \texttt{aab}\perp, \texttt{aabaa}\perp\}$ is $\{\texttt{a}, \texttt{b}\}$. The extension set of $\{\texttt{aaabb}\perp, \texttt{baa}\perp\}$ is also $\{\texttt{a}, \texttt{b}\}$.

The Patricia trie T on S is a tree where each node u carries a positional index $PI(u)$, and a representative pointer $RP(u)$. T can be recursively defined as follows:

1. If $|S| = 1$, then T has only one node whose its PI is $|S|$, and its RP references $s \in S$.
2. Otherwise, let σ be the LCS of S. The root of T is a node u with $PI(u) = |\sigma|$, and $RP(u)$ referencing s, where s is an arbitrary string in S.
3. Let E be the extension set of S. Then, u has $|E|$ child nodes, one for each character c in E. Specifically, the child node v_c for c is the root of a Patricia trie on the set of strings in S with $\sigma \cdot c$ as a prefix.

Example: Let $S = \{$aaabb$\perp$, aabaa$\perp$, aabab$\perp$, abbb$\perp$, abbba$\perp$, abbbb$\perp\}$. The Patricia trie of S is:



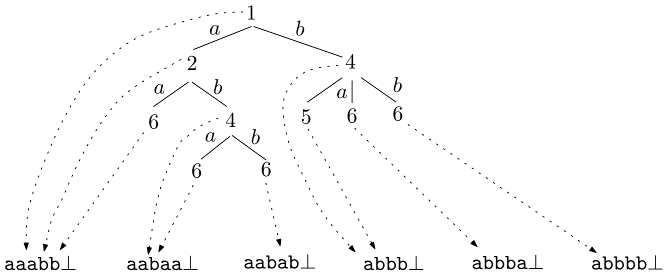
Lemma

A Patricia trie on m strings has at most $2m - 1$ nodes.

It is clear that every string in S corresponds to a leaf in its Patricia trie. Let u be a node in the Patricia trie. We say that $s \in S$ is in the subtree of u if the leaf corresponding to s is in the subtree of u.

Lemma

Let u be a node in a Patricia trie. Let $k = PI(u)$ and s the string referenced by $RP(u)$. All the strings in the subtree of u have prefix $s[1] \cdot s[2] \cdot ... \cdot s[k]$.



Think

How would you answer an exact matching query with $q =$ aabab⊥. How about $q =$ abbab⊥?

Combining the Patricia trie with the balanced trie, we obtain:

Theorem

For the exact matching problem on strings, there is a structure that occupies $O(m)$ space, and answers a query with string q in $O(\log m + |q|)$ time.

Think

How?

Let us now consider the prefix matching problem on strings:

Problem

Let S be a set of strings, each of which has a unique integer id. Given a query string q, a query reports all the ids of the strings $s \in S$ such that q is a prefix of S.

It is left as an exercise for you to design a structure that uses $O(m)$ space, and answers a query with string q in $O(\log m + |q| + k)$ time, where k is the number of ids reported.