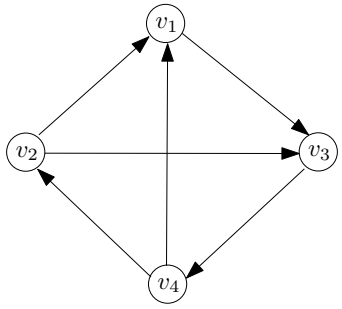
WST540: Exercise List 2

Problem 1. Consider the following graph:



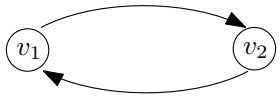
Let v_2 be the starting vertex in Google's random surfing model. Give the probabilities that v_1, v_2, v_3 and v_4 are the second vertex visited, respectively (recall that re-seeding happens with probability 15% at each step).

Solution. v_1 can be the second vertex in two scenarios:

- Re-seeding occurs right after the first step, and the re-seeding chooses v_2 (out of the four vertices). The probability is $15\% \times 25\% = 3.75\%$.
- Re-seeding does not occur, and the surfer chooses to follow the out-going link of v_2 to v_1 (between the two out-going links of v_2). The probability is $85\% \times 50\% = 42.5\%$.

Hence, the overall probability for v_1 to be the second vertex is $42.5\% + 3.75\% = 46.25\%$. In the same way, one can verify that the probabilities for v_2, v_3 and v_4 are 3.75%, 46.25% and 3.75%, respectively.

Problem 2. Consider the following graph:



Let v_1 be the starting vertex in Google's random surfing model. Give the probabilities that v_1 and v_2 are the i-th vertex visited respectively, for $i = 2, 3, ..., 5$.

Solution.

step	v_1	v_2
2	0.075	0.925
3	0.861	0.139
4	0.193	0.807
5	0.761	0.239

Problem 3*. What are the page ranks of v_1 and v_2 in the graph of Problem 2?

Solution. Suppose that the page rank of v_1 is x such that $0 \leq x \leq 1$. Then, the page rank of v_2 must be $1 - x$ (because the sum of their page ranks must be 1). Assume, without loss of generality,

that these values are obtained by running the power method using v_1 as the starting vertex. Then, if we run the power method using v_2 as the starting vertex instead, by symmetry, the algorithm will output that the page ranks of v_1 and v_2 are $1 - x$ and x, respectively.

Now we utilize the property that page ranks do *not* depend on the choice of the initial vertex. Therefore, $1 - x = x$ and hence, $x = 0.5$. It thus follows that the page ranks of v_1 and v_2 must both be 0.5.