## WST501: Final Exam

Problem 1 (30\%). Problem 1 of Exercise List 1.
Problem 2 (30\%). Problem 2 of Exercise List 3.
Problem 3 (40\%). In our discussion about using the R-tree to perform range search, the instructor mentioned that most existing R-tree update algorithms aim to create MBRs as square as possible. In this problem, we will gain more understanding about this heuristic. In particular, we will see that it leads to smaller expected cost if the query region is a square that distributes uniformly.

Consider a 2 d space where each dimension has a unit domain: $[0,1]$. Let $r$ be a rectangle with extents $a \times b$, where both $a$ and $b$ are in the range $[0,1]$. In other words, $r$ has length $a(b)$ on the x - ( y -) dimension. The center of $r$ coincides with the center of the universe. Consider $q$ to be a square with extents $\ell \times \ell$ satisfying:

- the center of $q$ distributes uniformly in the data space
- $a+\ell \leq 1$ and $b+\ell \leq 1$.

Prove that if the area of $r$ is fixed (i.e., $a b$ is a fixed value), the probability that $q$ intersects $r$ (they intersect if and only if they cover at least one common point in the data space) is minimized when $a=b$.

