WST501: Exercise List 3

Problem 1. Let us consider the classic *range search* problem. Let P be a set of n points in 2d space. Given an axis-parallel rectangle q, a query reports all the points in $P \cap q$. We can store P in a *kd-tree*, so that any query can be answered in $O(\sqrt{n} + k)$ time, where k is the number of points reported. We also know that the kd-tree can be constructed in $O(n \lg n)$ time.

Use the logarithmic method to obtain a semi-dynamic structure that answers a query in $O(\sqrt{n} \lg n + k)$ time, and supports an insertion in $O(\lg^2 n)$ amortized time.

Problem 2. Improve your solution to the previous problem so that a query is answered in $O(\sqrt{n}+k)$ time (i.e., no deterioration compared to the static structure), whereas the insertion cost remains unchanged.

Problem 3. Give a FIFO structure to solve the range search problem with query cost $O(\sqrt{n} + k)$, and amortized update cost $O(\lg^2 n)$, where the meanings of n and k are the same as in the previous problems.