Lecture Notes: R-trees

Yufei Tao
Chinese University of Hong Kong
taoyf@cse.cuhk.edu.hk

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In theoretical studies, we often develop structures that are *dedicated* to specific problems. In practice, often it is unrealistic to create many different indexes on the same dataset to support various types of queries, because doing so will incur prohibitive space consumption and update overhead. Therefore, it would be nice to have a single, all-around, structure, which occupies small space, can be updated efficiently, and most importantly, supports a large variety of queries.

In this lecture, we will discuss such an all-around structure called *the R-tree*. This structure is heuristic in nature, because it does not have any attractive theoretical guarantees on the search performance. Nevertheless, the practical efficiency of this structure has been widely established for many problems, especially if the dimensionality is low. Interestingly, for realistic datasets, there has been evidence [1] that R-trees even outperform some theoretical worst-case efficient structures. This is not as surprising as it may appear. The design of a theoretical structure aims at handling the most adverse datasets. Much of the design is not really needed for "good" datasets, and thus, may actually cause unnecessary overhead on such data. Because of its superb efficiency, the R-tree has become the *de facto* structure for multi-dimensional indexing in database systems nowadays.

1 The structure

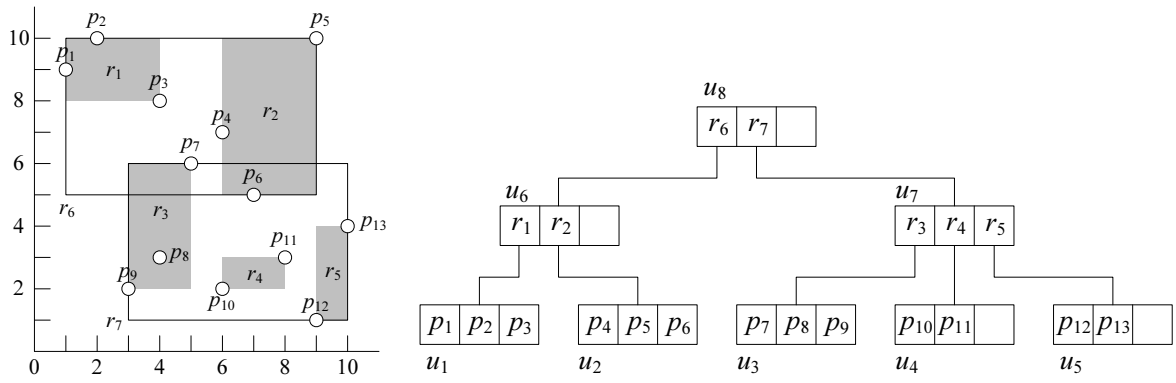
There are many variations of the R-tree in the literature [2, 4, 5, 6]. The version to be described below is similar to the *R^*-tree* [2], with some simplification in order to focus on the most crucial ideas. Also, our discussion is based on 2d point data, but the extensions to rectangle data and higher-dimensionalities are straightforward.

Let P be a set of points. An R-tree stores all these points in leaf nodes, each of which contains $\Theta(B)$ points, where B is the size of a disk page. Each non-leaf node u has $\Theta(B)$ children, except the root which must have 2 children at minimum unless it is the only node in the tree. For each child v, u stores a *minimum bounding rectangle* (MBR), which is the smallest rectangle that *tightly* encloses all the data points in the subtree of v. Note that there is no constraint on how points should be grouped into leaf nodes, and in general, how non-leaf nodes should be grouped into nodes of higher levels. Since each point is stored only once, the entire tree consumes linear space $O(N/B)$, where N is the cardinality of P.

Figure 1 shows an example where P has 13 points p_1, p_2, ..., p_{13}. Points p_1, p_2, p_3, for example, are grouped into leaf node u_1. This leaf is a child of of non-leaf node u_6, which stores an MBR r_1 for u_1. Note that r_1 tightly bounds all the points in u_1.

2 Insertions and deletions

Intuitively, in a good R-tree, nodes should have small MBRs. To see this intuitively, think about how to use the tree in Figure 1 to answer a range query. Namely, given a query rectangle q, we



(a) Data and MBRs (b) The structure

Figure 1: An R-tree

want to find all the points in P that are covered by q. It is easy to see that we only have to visit those nodes whose MBRs intersect q. Therefore, reducing the extents of the MBRs benefits query efficiency as fewer MBRs are expected to intersect q.

What do we mean, however, by *a small MBR*? Or in other words, what should the update algorithms of an R-tree minimize about the MBRs? A quick answer to think of is the area, but there exists a better answer. Figure 2 shows two MBRs, which actually have the same area. It turns out that the right (square) MBR leads to better performance in practice (where most range queries are square-like). Implication? We should minimize the *perimeter* of an MBR. This may look fairly reasonable in retrospect: a rectangle with a small perimeter almost always has a small area, but the opposite is not true.

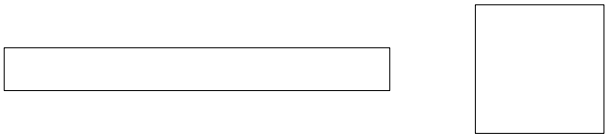


Figure 2: The right MBR is preferred

Next, we describe the insertion and deletion algorithms, both of which attempt to create square-like MBRs by reducing their perimeters as much as possible.

2.1 Insertions

To insert a point p, we use a strategy similar to that of a B-tree. Specifically, we add p to a leaf node u by following a single root-to-leaf path. If u overflows, split it, which creates a new child of $parent(u)$. In case $parent(u)$ overflows, also split it, which propagates upwards in the same manner. Finally, if the root is split, then a new root is created.

While all these sound familiar, there are, however, two important differences. First, although in the B-tree the insertion path is unique (i.e., the leaf supposed to accommodate the new item is unambiguous), this is not true at all for the R-tree. In fact, the new point p can be inserted into *any* leaf, which always results in a legal structure. If, however, a bad leaf is chosen (to contain p), its MBR may need to be enlarged substantially, thus harming the efficiency of the tree. Second, the split algorithm is not as trivial as in a B-tree because now we have multiple dimensions to tackle. Next, we will deal with the two issues separately. Note that the (heuristic) strategies to be

introduced are not the only ones. In fact, this is why there are so many variants of R-trees – each of them has its own strategies.

Choosing a subtree to insert. We are essentially facing the following problem. Given a non-leaf node u with children v_1, v_2, ..., v_f for some $f = \Theta(B)$, we need to pick the best child v^* such that the new point p is best inserted into the subtree of v^*. An approach that seems to work well in practice is a greedy one. Specifically, v^* can simply be the child v_i whose MBR requires the *least increase* of perimeter in order to cover p. For example, in Figure 3, both MBRs r_1 and r_2 must be expanded to enclose p, but r_2 incurs smaller perimeter increase, and hence, is a better choice.

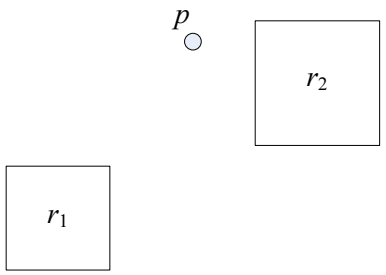


Figure 3: MBR r_2 requires smaller perimeter increase to cover p

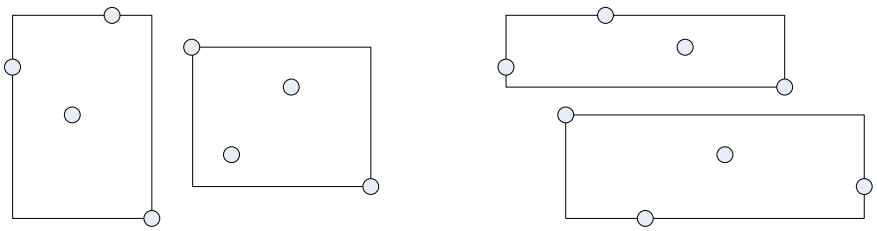
It is possible that p falls into the overlapping region of multiple MBRs. All those MBRs have a *tie* because none of them needs any perimeter increase to cover p. In this case, the winner can be decided according to other factors such as picking the MBR having the smallest area.

Node split. The node split problem can be phrased as follows. Given a set S of $B + 1$ points, split it into disjoint subsets S_1 and S_2 with $S_1 \cup S_2 = S$ such that

- $|S_1| \geq \lambda B$, $|S_2| \geq \lambda B$, where constant λ is the *minimum utilization rate* of a node, and
- the sum of the perimeters of $MBR(S_1)$ and $MBR(S_2)$ is small.

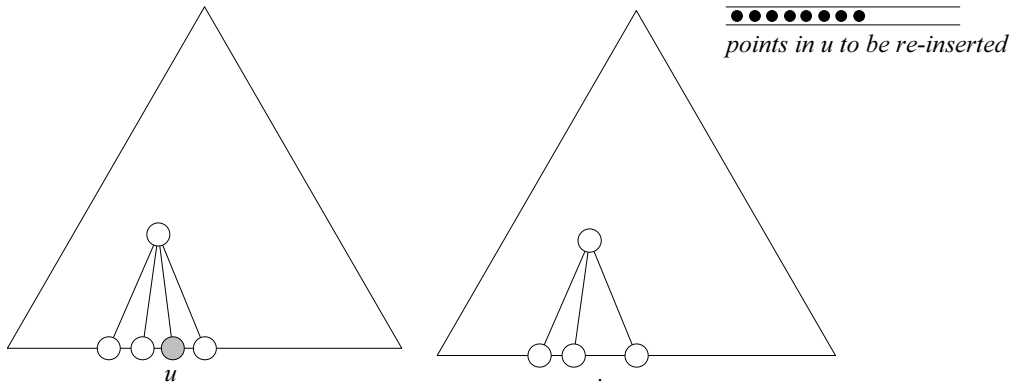
In the sequel, for simplicity we assume that $|S|$ is an even number, and $|S_1| = |S_2| = |S|/2$, i.e., we always aim at an even split. The extensions to uneven splits are straightforward.

Ideally, we would like to find the *optimal* split that minimizes the perimeter sum of $MBR(S_1)$ and $MBR(S_2)$. Since an MBR is decided by 4 coordinates (i.e., a pair of opposite corners), it is easy to find the optimal split in $O(B^4)$ time. This can be significantly improved to $O(B^2)$ time using a trick in [3]. Unfortunately, even a quadratic split time is usually excessively long in practice. Therefore, we turn our attention to heuristics that do not guarantee optimality, but usually produce fairly good splits. Next, we will describe a split algorithm that runs in $O(B \log B)$ time, or $O(dB \log B)$ time in general d-dimensional space.



(a) Split by cutting the x-dimension (b) Split by cutting the y-dimension

Figure 4: Splitting a node



(a) node u underflows (b) re-insertion

Figure 5: Handling a node underflow

The idea of our algorithm is to always split S using an axis-orthogonal cut. Consider, for example, a cut along the x-axis. For this purpose, we sort the points of S in ascending order of their x-coordinates. Then, we put the first $B/2$ points in the sorted order into S_1, and the rest into S_2. The split along the y-axis can be obtained in the same way. See Figure 4 (where S has 8 points). The final split is the better one of the two splits.

The above applies to splitting a leaf node. The case of non-leaf node is a bit different because the items to be split are MBRs, as opposed to points. Nevertheless, similar heuristics can still be applied by, for example, sorting the MBRs by their centrioids along each dimension.

2.2 Deletions

Deleting a point from an R-tree is carried out in an interesting manner. In particular, node underflows are handled in a way that differs considerably from the conventional merging approach as in a B-tree.

Specifically, let p be the point to be deleted. First, we need to find the leaf node u where p is stored. This can be achieved with a special range query using p itself as the search region. Then, p is removed from u. The deletion finishes if u still has λB items, where λ denotes the minimum node utilization. Otherwise, u *underflows*, which is handled by first removing u from its parent, and then *re-inserting* all the remaining points in u (using exactly the insertion algorithm mentioned earlier). See Figure 5.

Note that removing u from $parent(u)$ may cause $parent(u)$ to underflow too. In general, the underflow of a non-leaf node u' is also handled by re-insertions, with the only difference that the items re-inserted are MBRs, and each MBR is re-inserted to the same level of u'.

It is worth mentioning that while we can also design merging-based algorithms to handle node underflows, re-insertion actually gives better search performance [2]. This is because the structure of an R-tree is sensitive to the insertion order of the data points. Re-insertion gives the the early-inserted points to be inserted in other (better) branches, thus improving the overall structure.

References

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