INFS 4205/7205: Exercise Set 6

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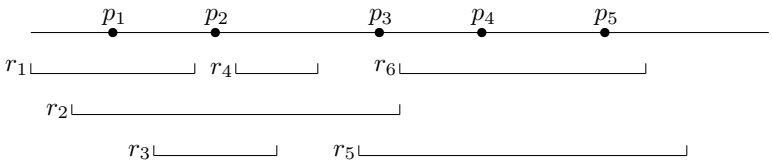
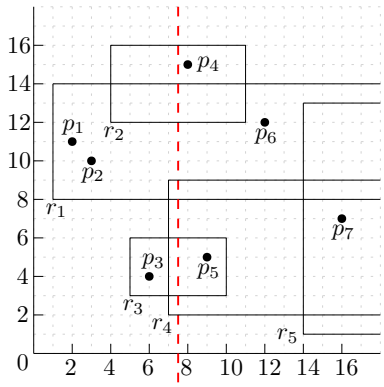
Problem 1. Consider the following instance of 1D *sorted* rectangles-join-points problem. Recall that we discussed an algorithm in the class, which maintains a linked list L as it processes the intervals and points from left to right. Answer the following questions:

- What are the contents right *before* the algorithm processes point p_3?
- What are the contents right *after* processing point p_3?



Problem 2. Consider the following 2D rectangles-join-points problem. As shown in the figure, the algorithm first divides the data space using the red line, and then recursively processes the left and the right of the line, respectively. Accordingly, in the recursion hierarchy, there are two child nodes of the root: corresponding to the processing on left and right of the line, respectively.

Let us focus on the right child of the root in the recursion hierarchy, where the algorithm processes (i) rectangles r_1, r_2, r_3, r_4, and r_5 (note that rectangles r_1, r_4, r_5 are "3-sided", namely, their right edges are aligned with the right boundary of the data space), and (ii) points p_4, p_5, p_6, and p_7. Here, the algorithm needs to construct a 1D instance of the rectangles-join-points problem. Indicate the intervals and points of the instance.



Problem 3. In the *segment join problem*, we are given a set H of horizontal segments, and a set V of vertical segments, both in $\mathbb{R}^2$. We want to report all pairs of $(h, r) \in H \times V$ such that h intersects r. Design an algorithm to do so in $O(n \log n + k)$ time, where $n = |H| + |V|$, and k is the number of pairs in the result.

Problem 4. In the *2D spatial join problem*, we are given two sets—R and S respectively—of axis-parallel rectangles in $\mathbb{R}^2$, and want to report all pairs $(r, s) \in R \times S$ such that r intersects s. Design an algorithm to do so in $O(n \log n + k)$ time, where $n = |R| + |S|$, and k is the number of

pairs in the result. Note that the running time should hold in the worst case (as opposed to the expected case).

Problem 5. In this problem, we extend the rectangles-join-points problem to arbitrary dimensionality d. We are given a set R of axis-parallel rectangles, and a set P of points, both in $\mathbb{R}^d$. The objective is to report all pairs $(r, p) \in R \times P$ such that rectangle r covers point p. Design an algorithm to do so in $O(n \log^{d-1} n + k)$ time, where $n = |R| + |P|$, and k is the number of pairs in the result.