# INFS 4205/7205: Exercise Set 5

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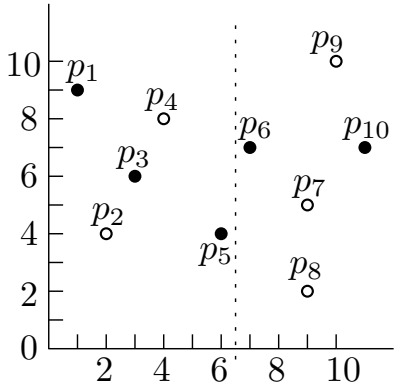
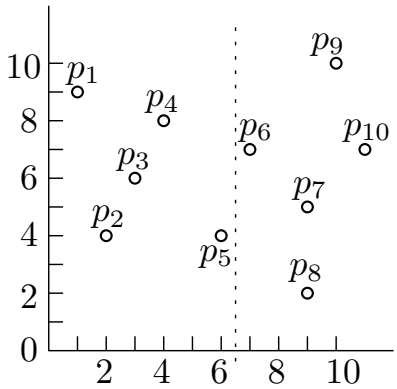
**Problem 1.** Consider that we run the divide-and-conquer algorithm to solve the dominance screening on the following 2D dataset (i.e., report the white points that are not dominated by the black ones). The algorithm divides the dataset along the x-dimension in halves, where the first half consists of points $p_1, p_2, p_3, p_4, p_5$, and the second consists of $p_6, p_7, p_8, p_9, p_{10}$. Answer the following questions:

- What is the answer set returned from each half?
- In order to merge the two answer sets into the final answer, the algorithm needs to solve a 1D dominance screening instance. Indicate the 1D points (by specifying their coordinates) in this instance.




**Problem 2.** Consider that we run the divide-and-conquer algorithm to find the skyline on the following dataset (i.e., report all the points that are not dominated by any other point). The algorithm divides the dataset along the x-dimension in halves, where the first half consists of points $p_1, p_2, p_3, p_4, p_5$, and the second consists of $p_6, p_7, p_8, p_9, p_{10}$. Answer the following questions:

- What is the answer set returned from each half?
- In order to merge the two answer sets into the final answer, the algorithm needs to solve a 1D dominance screening instance. Indicate the 1D points (by specifying their coordinates) in this instance.



**Problem 3.** Consider that we run the divide-and-conquer algorithm to find the skyline on the following 3D dataset:

$$p_1 = (1, 2, 3), p_2 = (2, 4, 6), p_3 = (4, 7, 9), p_4 = (4, 9, 3), p_5 = (6, 3, 1),$$
$$p_6 = (7, 2, 9), p_7 = (8, 4, 5), p_8 = (8, 6, 7), p_9 = (9, 9, 8), p_{10} = (9, 2, 2).$$

The algorithm divides the dataset along the x-dimension in halves, where the first half consists of points $p_1, p_2, p_3, p_4, p_5$, and the second consists of $p_6, p_7, p_8, p_9, p_{10}$. Answer the following questions:

- What is the answer set returned from each half?
- In order to merge the two answer sets into the final answer, the algorithm needs to solve a 2D dominance screening instance. Indicate the 2D points (by specifying their coordinates) in this instance.

**Problem 4.** Consider the 2D dominance screening problem again. Recall that we have two sets $P, Q$ of points in $\mathbb{R}^2$. The goal is to identify all the points $q \in Q$ such that $q$ is not dominated by any point in $P$. Suppose that $P$ and $Q$ have already been sorted by x-dimension. Give an algorithm to solve the problem in $O(n)$ time where $n = |P| + |Q|$. You may assume that all the points in $P \cup Q$ have distinct x-coordinates.

**Problem 5*.** Consider the 3D dominance screening (DS) problem. Recall that we have two sets $P, Q$ of points in $\mathbb{R}^3$. The goal is to identify all the points $q \in Q$ such that $q$ is not dominated by any point in $P$. Give an algorithm to solve the problem in $O(n \log n)$ time where $n = |P| + |Q|$. Again, you may assume that all the points in $P \cup Q$ have distinct x-coordinates.

**Problem 6.** Give an algorithm to find the skyline of $n$ points in $\mathbb{R}^d$ (where $d \geq 3$) in $O(n \log^{d-2} n)$ time.