Problem 1. You are given a positive integer $n$ (that is stored in a register of the CPU). Design an algorithm to determine whether $n$ is an even number. Your algorithm should have a cost no more than 10.

Problem 2. You are given two positive integers $n$ and $m$ (stored in two registers of the CPU). Design an algorithm to calculate $n \mod m$. Your algorithm should have a cost no more than 10.

Note: $n \mod m$ is the “remainder” of $n$ divided by $m$. For example, $10 \mod 2 = 0$ and $13 \mod 3 = 1$.

Problem 3. You are given a positive integer $n$ (that is stored in a register of the CPU). Design an algorithm to determine whether $n$ is a prime number. Your algorithm should have a cost no more than $100\sqrt{n}$.

Problem 4. You are given two positive integers $n$ and $m$ (stored in two registers of the CPU), where $n$ is a power of 2. Design an algorithm to calculate $m^n$. Your algorithm should have a cost no more than $100 \log_2 n$.

Problem 5. You are given two sets $S_1$ and $S_2$ of integers. Specifically, $|S_1| = n$ (that is, the number of integers in $S_1$—the size of $S_1$—is $n$) while $|S_2| = m$. The integers in $S_1$ and $S_2$ have been stored in memory as shown in the figure below. In particular, the integers in $S_1$ have been sorted in ascending order, while those in $S_2$ have not. The starting address $x$ of $S_1$ and the starting address $y$ of $S_2$ have been stored in the CPU. So are the values of $n$ and $m$.

Design an algorithm to determine whether $S_1 \cap S_2$ is empty—in other words, whether the two sets have a common integer. Your algorithm should have a cost no more than $100m \log_2 n$. 