Problem 1. Which of the following is not an atomic operation of the RAM model?
Answer: [ ]
A. Calculate $a + b$ where $a$ and $b$ are stored in two registers.
B. Sort an array of $n$ integers for an arbitrary value of $n$.

Answer: B

Problem 2. Which of the following is true? Answer: [ ]
A. $n \log_2 n = O(n)$.  
B. $n = O(n \log n)$.

Answer: B

Problem 3. Which of the following is true? Answer: [ ]
A. $n^2 = \Omega(n)$.  
B. $n = \Omega(n^2)$.

Answer: A

Problem 4. Which of the following is true? Answer: [ ]
A. $100n + \sqrt{n} + (\log_2 n)^5 = \Theta(n)$.  
B. $100n + \sqrt{n} + (\log_2 n)^5 \neq \Theta(n)$.

Answer: A

Problem 5. Which of the following functions of $n$ is not $O(n^2)$. Answer: [ ]
A. $n^2 / \log^2 n$  
B. $(\log_2 n)^{35}$  
C. 100000  
D. $n^{2.001} / 3583$  
E. $n^3 / 2\pi$

Answer: D

Problem 6. Which of the following functions of $n$ is $O(n)$. Answer: [ ]
A. 100000  
B. $n^2 / \log^2 n$  
C. $n^{2.001} / 3583$  
D. $(1.01)^n$  
E. $n \log_2 n$

Answer: A

Problem 7. Which of the following functions of $n$ is $\Omega(n)$. Answer: [ ]
A. 100000  
B. $n / \log^2 n$  
C. $n^{0.999}$  
D. $(1.01)^n$  
E. $(\log_2 n)^{35}$

Answer: D

Problem 8. Which of the following functions of $n$ is not $\Theta(n \log n)$. Answer: [ ]
A. $35n \log_2 n + \sqrt{n}$  
B. $n \log_{35} n$  
C. $n^{1.81} + n \log_2 n$  
D. $n^{0.99} + 87n \log_{200} n$.

Answer: C

Problem 9. Which of the following statements is true? Answer: [ ]
A. The running time of binary search (performed on a sorted array of \(n\) integers) is \(O(\log^2 n)\).
B. The running time of binary search (performed on a sorted array of \(n\) integers) is \(O(1)\).
C. \(35n + \sqrt{n} = \Theta(n^2)\).
D. In the RAM model, the time complexity of an algorithm depends on how fast a CPU is (the complexity on a 2 GHz CPU may be different from that on a 1 GHz one).

Answer: A

Problem 10. Prove \(10n + n^{1/3} = O(n)\).

Answer: Set \(c_1 = 11\) and \(c_2 = 1\). The inequality \(10n + n^{1/3} \leq c_1n\) holds for all \(n \geq c_2\). This completes the proof.
COMP7505: Quiz 1

Name:
Student ID:

This is the quiz paper for COMP7505. If you are registered for COMP3506, turn overleaf.

Problems 1-8 bear 10 marks each. Problem 9 bears 20 marks.

Problem 1. Which of the following is true? Answer: [ ]
A. \( n \log_2 n = O(n) \).  
B. \( n = O(n \log n) \).

Answer: B

Problem 2. Which of the following is true? Answer: [ ]
A. \( 100n + \sqrt{n} + (\log_2 n)^5 = \Theta(n) \).  
B. \( 100n + \sqrt{n} + (\log_2 n)^5 \neq \Theta(n) \).

Answer: A

Problem 3. Which of the following functions of \( n \) is not \( O(n^2) \). Answer: [ ]
A. \( n^2 / \log^2 n \)  
B. \( (\log_2 n)^{35} \)  
C. 100000  
D. \( \frac{\pi^{2.001}}{3583} \)  
E. \( n^{\frac{3}{2} \pi} \)

Answer: D

Problem 4. Which of the following functions of \( n \) is \( O(n) \). Answer: [ ]
A. 100000  
B. \( n^2 / \log^2 n \)  
C. \( \frac{n^{32.001}}{3583} \)  
D. \( (1.01)^n \)  
E. \( n \log_2 n \)

Answer: A

Problem 5. Which of the following functions of \( n \) is \( \Omega(n) \). Answer: [ ]
A. 100000  
B. \( n / \log^2 n \)  
C. \( n^{0.999} \)  
D. \( (1.01)^n \)  
E. \( (\log_2 n)^{35} \)

Answer: D

Problem 6. Which of the following functions of \( n \) is not \( \Theta(n \log n) \). Answer: [ ]
A. \( 35n \log_2 n + \sqrt{n} \)  
B. \( n \log_{35} n \)  
C. \( n^{1.81} + n \log_2 n \)  
D. \( n^{0.99} + 87n \log_{200} n \)

Answer: C

Problem 7. Which of the following statements is true? Answer: [ ]
A. The running time of binary search (performed on an array of \( n \) integers) is \( O(\log^2 n) \).
B. The running time of binary search (performed on an array of \( n \) integers) is \( O(1) \).
C. \( 35n + \sqrt{n} = \Theta(n^2) \).
D. In the RAM model, the time complexity of an algorithm depends on how fast a CPU is (the complexity on a 2 GHz CPU may be different from that on a 1 GHz one).

Answer: A

Problem 8. Prove \( 10n + n^{1/3} = O(n) \).

Answer: Set \( c_1 = 11 \) and \( c_2 = 1 \). The inequality \( 10n + n^{1/3} \leq c_1 n \) holds for all \( n \geq c_2 \). This completes the proof.
Problem 9. Let $f(n)$ and $g(n)$ be two functions of integer $n$. Prove: if $f(n) = O(g(n))$, then $\frac{f(n)}{g(n)} = O(1)$.

Answer: As $f(n) = O(g(n))$, there exist constants $c_1, c_2$ such that $f(n) \leq c_1 \cdot g(n)$ for all $n \geq c_2$. This means $\frac{f(n)}{g(n)} \leq c_1$ for all $n \geq c_2$, namely, $f(n)/g(n) = O(1)$. 
