# The RAM Computation Model

Yufei Tao

ITEE
University of Queensland

This is not a programming course.

**Take-away message 1 from this course**

Programming is the last step in software development, which occurs only after the algorithms are clear.

**Take-away message 2**

No programming knowledge is needed to study algorithms, which is the core of computer science: a beautiful branch of mathematics.

Many people believe that this branch holds the future of mankind.

In mathematics (and hence, computer science) everything—including every term and symbol—must be rigorous.

Computer science is a subject where we

1. first define a **computation model**, which is a simple yet accurate abstraction of a computing machine;
2. then slowly build up a theory in this model from scratch.

## The Random Access Machine (RAM) model

A machine has a **CPU** and a **memory**.

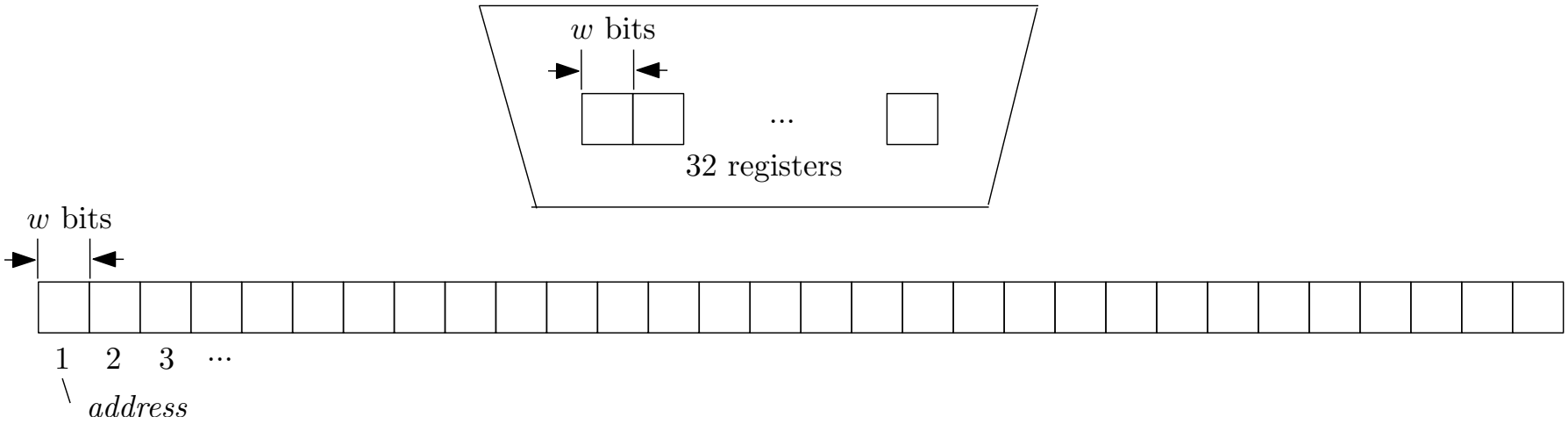
### Memory

- An infinite sequence of **cells**, each of which contains the same number $w$ of bits.
- Every cell has an **address**: the first cell of memory has address 1, the second cell 2, and so on.

## The Random Access Machine (RAM) model

### CPU

- Contains a fixed number—32 in this course—of **registers**, each of which has $w$ bits (i.e., same as a memory cell).



## The Random Access Machine (RAM) model

### CPU

- Can do the following **atomic operations**:
  1. **(Register (Re-)Initialization)**
     Set a register to a fixed value (e.g., 0, $-1$, 100, etc.), or to the contents of another register.

## The Random Access Machine (RAM) model

### CPU

- Can do the following 4 **atomic operations**:

  2. **(Arithmetic)**
     Take the integers $a, b$ stored in two registers, calculate one of the following and store the result in a register:
     - $a + b$, $a - b$, $a \cdot b$, and $a/b$.

> Note: $a/b$ is "integer division", which returns an integer.
> For example, $6/3 = 2$ and $5/3 = 1$.

## The Random Access Machine (RAM) model

### CPU

- Can do the following 4 **atomic operations**:

  3. **(Comparison/Branching)**
     Take the integers $a, b$ stored in two registers, compare them, and learn which of the following is true:
     - $a < b$, $a = b$, $a > b$.

## The Random Access Machine (RAM) model

### CPU

- Can do the following 4 **atomic operations**:

  4. **(Memory Access)**
     Take a memory address $A$ currently stored in a register. Do one of the following:
     - Read the contents of the memory cell with address $A$ into another register (overwriting the bits there).
     - Write the contents of another register into the memory cell with address $A$ (overwriting the bits there).

## The Random Access Machine (RAM) model

### Algorithms and Their Cost

An **algorithm** is a sequence of atomic operations.

Its **cost** (also called its **running time**, or simply, **time**) is the length of the sequence, namely, the number of atomic operations.

## The Random Access Machine (RAM) model

A **word** is a sequence of $w$ bits, where $w$ is called the **word length**.

- In other words, each memory cell and CPU register store a word.

Unless otherwise stated, you do not need to pay attention to the value of $w$ in this course.

Example 1

Problem

Suppose that an integer of $n \geq 1$ has already been stored at the memory cell of address 1. We want to calculate $1 + 2 + ... + n$ (the sum can be stored anywhere, e.g., in a register).

Example 1

Algorithm

Set register $b$ to 0, $c$ to 1, and $d$ to 1. Load $n$ into register $a$.

Repeat the following until $c > a$:

- Calculate $b + c$, and store the result in $b$.
- Calculate $c + d$, and store the result in $c$ (effectively increasing $c$ by 1).

Cost of the algorithm $= 3n + 4$

Think: which atomic operations are performed?

We have described the algorithm essentially in English words. The next slide shows a different way to describe the same algorithm.

## Example 1

### Algorithm

1. load $n$ into register $a$
2. register $b \leftarrow 0$, $c \leftarrow 1$, $d \leftarrow 1$
3. **repeat**
4. $\quad b \leftarrow b + c$
5. $\quad c \leftarrow c + d$
6. **until** $c > a$
7. **return** $b$

The above is called pseudocode. As you can see, we are not restricting ourselves to any particular programming languages. The description is in a "free form", mixing English words and "programming-like" statements as we wish, as long as it serves the purpose of conveying—without ambiguity—how our algorithm runs.

Example 2

Same problem as before.

Problem

Suppose that an integer of $n \geq 1$ has already been stored at the memory cell of address 1. We want to calculate $1 + 2 + ... + n$ (the sum can be stored anywhere, e.g., in a register).

But this time we will give a faster algorithm.

Example 2

Algorithm

Set register $a$ to 1, and $b$ to 2. Load $n$ into register $c$. Then:

- Set $a$ to $a + c$ (note: $a$ now equals $n + 1$).
- Set $a$ to $a * c$ (now equals $n(n + 1)$).
- Set $a$ to $a/b$ (now equals $n(n + 1)/2$).

Cost of the algorithm $= 6$

This is significantly faster than the previous algorithm when $n$ is large. In particular, the time of the previous algorithm increases linearly with $n$, while the time of the above one remains constant.

Although we have not talked about how to implement our algorithms into actual programs, now it should be straightforward for you (who must have learned a programming language prior to taking this course) to do so.

Indeed, to design algorithms, a real computer science person never has a programming language in mind. Instead, s/he focuses on the simple abstraction of the RAM model, knowing that once an algorithm is out, implementing it into an actual program is merely a matter of translation and experience.