CMSC5724: Quiz 3

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Problem 1 Solution:

Iteration 1. Let $o_1 = a, o_2 = d$, and $o_3 = g$. The algorithm divides P into partitions P_1, P_2 and P_3 such that P_i ($1 \leq i \leq 3$) includes all the points in P with o_i as their closest centroids. Specifically, $P_1 = \{a, b, c\}$, $P_2 = \{d, e, h, i, j\}$, and $P_3 = \{f, g\}$. Then, the algorithm resets o_i to the geometric centroid of P_i: $o_1 = (\frac{8}{3}, 3)$, $o_2 = (6, 7)$, and $o_3 = (9, 5)$.

Iteration 2. The algorithm re-divides P into P_1, P_2 and P_3 based on the current centroids: $P_1 = \{a, b, c\}$, $P_2 = \{h, i, j\}$, and $P_3 = \{d, e, f, g\}$. Accordingly, the centroids are re-computed as $o_1 = (\frac{8}{3}, 3)$, $o_2 = (\frac{16}{3}, \frac{26}{3})$, and $o_3 = (8, \frac{19}{4})$.

Iteration 3. We get $P_1 = \{a, b, c\}$, $P_2 = \{h, i, j\}$, and $P_3 = \{d, e, f, g\}$ again after re-dividing P based on the current centroids. The algorithm terminates.

Problem 2 Solution: The covariance matrix is $\mathbf{A} = \begin{bmatrix} 2/3 & 2/3 \\ 2/3 & 2/3 \end{bmatrix}$. This matrix has two Eigenvalues: $\lambda_1 = 4/3$ and $\lambda_2 = 0$. To return a 1-space, the PCA finds a unit eigenvector of λ_1: such a vector is $\boldsymbol{u}_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$. Therefore, the 1-space returned by PCA is the span of $\boldsymbol{u}_1$.

The projections of the three points of P onto the 1-space are: $(-1, -1), (0, 0), (1, 1)$.