CMSC5724: Quiz 2

Name: **Student ID:**

Solution to Problem 1.

$$\begin{aligned} & (\phi(\vec{p}) + \phi(\vec{q})) \cdot (\phi(\vec{p}) - \phi(\vec{q})) \\ = \; & \phi(\vec{p}) \cdot \phi(\vec{p}) - \phi(\vec{q}) \cdot \phi(\vec{q}) \\ = \; & K(\vec{p}, \vec{p}) - K(\vec{q}, \vec{q}) \end{aligned}$$

Solution to Problem 2. Suppose that there exists a set P satisfying VC-dim$(P, \mathcal{H}') >$ VC-dim$(P, \mathcal{H})$. Then, there must be a subset S of P such that $|S| =$ VC-dim$(P, \mathcal{H}')$ and S is shattered by $\mathcal{H}'$. According to the fact $\mathcal{H}' \subseteq \mathcal{H}$, we know S is also shattered by $\mathcal{H}$. However, this implies that VC-dim$(P, \mathcal{H}') \leq$ VC-dim$(P, \mathcal{H})$, thus causing a contradiction.

Solution to Problem 3. Let $W = \{w_1, w_2, w_3\}$.

$$\begin{aligned} margin(A \mid W) &= \min\{\frac{\boldsymbol{w_2} \cdot \boldsymbol{A} - \boldsymbol{w_1} \cdot \boldsymbol{A}}{\sqrt{2 \times \sum_{i=1}^{3} |\boldsymbol{w_i}|^2}}, \frac{\boldsymbol{w_2} \cdot \boldsymbol{A} - \boldsymbol{w_3} \cdot \boldsymbol{A}}{\sqrt{2 \times \sum_{i=1}^{3} |\boldsymbol{w_i}|^2}}\} = \min\{\frac{5-(-6)}{\sqrt{14}}, \frac{5-(-2)}{\sqrt{14}}\} \\ &= \frac{7}{\sqrt{14}} \end{aligned}$$

Similarly,

$$margin(B \mid W) = \min\{\frac{2-(-4)}{\sqrt{14}}, \frac{2-0}{\sqrt{14}}\} = \frac{2}{\sqrt{14}}$$

$$margin(C \mid W) = \min\{\frac{6-(-2)}{\sqrt{14}}, \frac{6-1}{\sqrt{14}}\} = \frac{5}{\sqrt{14}}$$

$$margin(D \mid W) = \min\{\frac{4-1}{\sqrt{14}}, \frac{4-(-3)}{\sqrt{14}}\} = \frac{3}{\sqrt{14}}$$

Therefore, the classifier's margin equals $\frac{2}{\sqrt{14}}$.