## CMSC5724: Exercise List 12

Problem 1. In the following directed graph $G$, every node represents a webpage, and every edge represents a hyperlink. Consider the "Google random surfing" model with parameter $\alpha=1 / 2$. Recall that the model can be regarded as a random walk on a complete graph, where each edge is attached a transition probability. Show this complete graph and give all the transition probabilities.


Problem 2. Compute the exact page rank of every node in problem 1.
Problem 3. Define $r_{i}$ as the page rank of $v_{i}$ in problem 2; and let $P=\left(r_{1}, r_{2}, r_{3}, r_{4}\right)^{T}$. Use the power method to compute an approximate page rank for every node. Show all the steps of the power method until $\operatorname{Err}(t) \leq 0.01$ (see the definition of $\operatorname{Err}(t)$ in our lecture notes).

Problem 4. Consider a new definition similar to $\operatorname{Err}(t)$ :

$$
\operatorname{Err}^{\prime}(t)=\max _{i=1}^{n}\left|r_{i}-P\left(v_{i}, t\right)\right|
$$

where the meanings of $r_{i}$ and $P\left(v_{i}, t\right)$ are the same as in Slide 22 of the lecture notes. Prove that, for any $0<\epsilon \leq 1$, the power method ensures $\operatorname{Err}^{\prime}(t) \leq \epsilon$ after $t=O\left(\log \frac{1}{\epsilon}\right)$ rounds.

