CMSC5724: Quiz 3

Hand-write all your solutions on paper. Take a picture of the paper together with your CUHK student ID card. Upload the picture to Blackboard or email it to the instructor at taoyf@cse.cuhk.edu.hk. Your must do so within 20 minutes after the quiz has started.

Problem 1 (40%). Consider the kernel function $K(p,q) = (2(p \cdot q) + 1)^2$, where $p = (p[1], p[2])$ and $q = (q[1], q[2])$ are 2D vectors. Recall that there is a mapping function $\phi$ from $\mathbb{R}^2$ to $\mathbb{R}^d$ for some integer $d$, such that $K(p,q)$ equals the dot product of $\phi(p)$ and $\phi(q)$. Give the details of $\phi$.

Answer: Rewrite $K$ as dot product form.


Problem 2 (10%). Consider a 3-class linear classifier in 2D space that is defined by vectors $w_1 = (3, 5)$, $w_2 = (-2, 9)$, and $w_3 = (0, 7)$. Given a point $p = (-5, 1)$, explain what is the label assigned to $p$ and why.

Answer: Computing the dot product between each $w_i$ and $p$ where $i \in [1, 3]$, we have:

- $w_1 \cdot p = -10$;
- $w_2 \cdot p = 19$;
- $w_3 \cdot p = 7$.

Since $w_2 \cdot p$ is largest, the label assigned to $p$ is 2.

Problem 3 (50%). In the lecture, we proved that the $k$-center algorithm is $2$-approximate. In this problem, you will see that the approximation ratio 2 is tight. Consider the $k$-center problem on the following set $P$ of one-dimensional points (the numbers indicate coordinates):

Answer the following questions for $k = 2$:

1. What is the optimal set $C^*$ of centroids? What is the radius of $C^*$ (namely, $r(C^*)$, using the notations in the lecture notes)?

2. Prove: the $k$-center algorithm always returns a centroid set whose radius is $2 \cdot r(C^*)$.

Answer: 1. $C^* = \{b, e\}$ and $r(C^*) = 1$.

2: Let $C = \{o_1, o_2\}$ be the set returned by the $k$-center algorithm. Assume that $o_1$ (or $o_2$, resp.) is the first (or the second, resp.) point added into $C$.

When $o_1 \in \{a, b, c\}$, $o_2$ must be $f$. We have $r(C) = 2$.

When $o_1 \in \{d, e, f\}$, $o_2$ must be $a$. We also have $r(C) = 2$.

Therefore, the radius of the centroid set returned by the $k$-center algorithm is always $2 \cdot r(C^*)$. 