Problem 1. What is the margin of the separation line $\ell : -x - 5y = 0$?

Problem 2. Run Margin Perceptron on $P$ with $\gamma_{guess} = 0.1$, and give the equation of the line that is maintained by the algorithm at the end of each iteration.

Problem 3. Same as the previous problem but with $\gamma_{guess} = 4/\sqrt{26}$.

Problem 4. Give an instance of quadratic programming to find an origin-passing separation plane with the maximum margin.

Problem 5. Consider the following instance of quadratic programming in $\mathbb{R}^d$:

$$\begin{array}{l}
\text{minimize } |w| \\
\text{subject to } \\
\quad w \cdot p_i \geq 1 \text{ for each } i \in [1,n]
\end{array}$$

where $p_1, \ldots, p_n$ are $n$ given points in $\mathbb{R}^d$. Prove: if an optimal $w$ exists, there must exist at least one $i \in [1,n]$ such that $w \cdot p_i = 1$.

Problem 6. Let $\gamma_{opt}$ be the maximum margin of an origin-passing separation plane on a set $P$ of points. Denote by $R$ the largest distance from a point in $P$ to the origin.

Suppose that, given a value $\gamma$, margin Perceptron ensures the following:

- if it terminates, it definitely returns a separation plane with margin at least $\alpha \cdot \gamma$, where $\alpha$ is an arbitrary constant less than 1;
- if $\gamma \leq \gamma_{opt}$, it definitely terminates after at most $c \cdot R^2/\gamma^2$ corrections, for some constant (which depends on $\alpha$).

Design an algorithm to find a separation plane with margin at least $\alpha \cdot \beta \cdot \gamma_{opt}$ after $O(R^2/\gamma_{opt}^2)$ corrections in total, where $\beta$ can be any constant less than 1.