CMSC5724: Exercise List 2

Answer the following questions based on the training set below

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>class</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>−</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>−</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>2</td>
<td>1</td>
<td>0</td>
<td>−</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>−</td>
</tr>
</tbody>
</table>

**Problem 1.** Estimate \( \Pr[\text{class} = +] \) and \( \Pr[C = 2 \mid \text{class} = +] \).

**Answer.** Since 5 out of 10 records belong to the class +, we estimate \( \Pr[\text{class} = +] \) to be 1/2. Among the 5 records of class +, 2 of them have \( C = 2 \). Hence, we estimate \( \Pr[C = 2 \mid \text{class} = +] \) to be 2/5.

**Problem 2.** Let us make the following conditional independence assumption: conditioned on a specific class, attributes \( A, B, C \) are independent. Estimate \( \Pr[A = 1, B = 2 \mid \text{class} = +] \).

**Answer.** Under the assumption we have:

\[
\Pr[A = 1, B = 2 \mid \text{class} = +] = \Pr[A = 1 \mid \text{class} = +] \cdot \Pr[B = 2 \mid \text{class} = +]
\]

We estimate \( \Pr[A = 1 \mid \text{class} = +] = 3/5 \) and \( \Pr[B = 2 \mid \text{class} = +] = 2/5 \). Therefore, our estimate of \( \Pr[A = 1, B = 2 \mid \text{class} = +] \) is 6/25.

**Problem 3.** Under the conditional independence assumption, decide the larger probability between \( \Pr[\text{class} = + \mid A = 1, B = 2, C = 0] \) and \( \Pr[\text{class} = - \mid A = 1, B = 2, C = 0] \).

**Answer.** By Bayes Theorem

\[
\Pr[\text{class} = + \mid A = 1, B = 2, C = 0] = \frac{\Pr[A = 1, B = 2, C = 0 \mid \text{class} = +] \cdot \Pr[\text{class} = +]}{\Pr[A = 1, B = 2, C = 0]}
\]

and

\[
\Pr[\text{class} = - \mid A = 1, B = 2, C = 0] = \frac{\Pr[A = 1, B = 2, C = 0 \mid \text{class} = -] \cdot \Pr[\text{class} = -]}{\Pr[A = 1, B = 2, C = 0]}
\]

To compare the two, we only need to estimate the numerators of the above fractions. Towards this purpose, we have:

\[
\text{(estimate)} = \frac{3 \cdot 2 \cdot 1}{5 \cdot 5 \cdot 5} \cdot \frac{1}{2} = \frac{6}{250}
\]
\[
\Pr[A = 1, B = 2, C = 0 \mid class = -] \cdot \Pr[class = -] \\
= \Pr[A = 1 \mid class = -] \cdot \Pr[B = 2 \mid class = -] \cdot \Pr[C = 0 \mid class = -] \cdot \Pr[class = -]
\]

(estimate) \[= \frac{5}{25} \cdot \frac{3}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{3}{250}
\]

We therefore conclude that \( \Pr[class = + \mid A = 1, B = 2, C = 0] > \Pr[class = - \mid A = 1, B = 2, C = 0] \).

**Problem 4.** Suppose that we make an alternative conditional independence assumptions: \( A \) and \( C \) are independent, conditioned on a class label and a value of \( B \). Decide the larger probability between \( \Pr[class = + \mid A = 1, B = 2, C = 0] \) and \( \Pr[class = - \mid A = 1, B = 2, C = 0] \).

**Answer.** By Bayes Theorem

\[
\Pr[class = + \mid A = 1, B = 2, C = 0] \\
= \frac{\Pr[A = 1, B = 2, C = 0 \mid class = +] \cdot \Pr[class = +]}{\Pr[A = 1, B = 2, C = 0]}
\]

\[
= \frac{\Pr[A = 1, C = 0 \mid class = +, B = 2] \cdot \Pr[B = 2 \mid class = +] \cdot \Pr[class = +]}{\Pr[A = 1, B = 2, C = 0]}
\]

\[
= \frac{\Pr[A = 1 \mid class = +, B = 2] \cdot \Pr[C = 0 \mid class = +, B = 2] \cdot \Pr[B = 2 \mid class = +] \cdot \Pr[class = +]}{\Pr[A = 1, B = 2, C = 0]}
\]

where the last step used the given conditional independence assumption. Likewise,

\[
\Pr[class = - \mid A = 1, B = 2, C = 0] \\
= \frac{\Pr[A = 1 \mid class = -, B = 2] \cdot \Pr[C = 0 \mid class = -, B = 2] \cdot \Pr[B = 2 \mid class = -] \cdot \Pr[class = -]}{\Pr[A = 1, B = 2, C = 0]}
\]

Now we compare the numerators of the two fractions:

\[
\Pr[A = 1 \mid class = +, B = 2] \cdot \Pr[C = 0 \mid class = +, B = 2] \cdot \\
\Pr[B = 2 \mid class = +] \cdot \Pr[class = +]
\]

(estimate) \[= \frac{1}{2} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{\gamma}{10}
\]

where \( \gamma > 0 \) is a very small value introduced to avoid probability 0.

\[
\Pr[A = 1 \mid class = -, B = 2] \cdot \Pr[C = 0 \mid class = -, B = 2] \cdot \\
\Pr[B = 2 \mid class = -] \cdot \Pr[class = -]
\]

(estimate) \[= \gamma \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{\gamma^2}{10}
\]

We thus conclude that \( \Pr[class = + \mid A = 1, B = 2, C = 0] > \Pr[class = - \mid A = 1, B = 2, C = 0] \).