Problem 1. Consider table $T(A, B, C, D, E)$ with candidate key $AB$. Prove or disprove: function dependency $AB \rightarrow C$ holds on $T$.

Answer. Yes. By definition, $AB \rightarrow C$ requires that if two tuples $t_1, t_2$ satisfy $t_1[A] = t_2[A]$ and $t_1[B] = t_2[B]$, then $t_1[C] = t_2[C]$. Since $AB$ is a candidate key, $t_1[A] = t_2[A]$ and $t_1[B] = t_2[B]$ imply that $t_1 = t_2$. Therefore, $t_1[C] = t_2[C]$ definitely holds.

Problem 2. Consider table $T(A, B, C, D, E)$ with candidate key $AB$. Prove or disprove: function dependency $ABC \rightarrow D$ holds on $T$.

Answer. Yes. By definition, $ABC \rightarrow D$ requires that if two tuples $t_1, t_2$ satisfy $t_1[A] = t_2[A]$, $t_1[B] = t_2[B]$ and $t_1[C] = t_2[C]$, then $t_1[D] = t_2[D]$. Since $AB$ is a candidate key, $t_1[A] = t_2[A]$ and $t_1[B] = t_2[B]$ imply that $t_1 = t_2$. Therefore, $t_1[D] = t_2[D]$ definitely holds (note that we do not even need the condition $t_1[C] = t_2[C]$ to derive this).

Problem 3. Consider table LOAN(cid, lid, bid, amount), where each tuple describes a loan. Specifically, $cid$ is the id of the customer borrowing the loan, $lid$ is the id of the loan itself, $bid$ is the id of the branch lending the loan, and $amount$ is the loan amount. Give a functional dependency to enforce each of the following constraints:

(i) Every customer can borrow only one loan.
(ii) No loan can be borrowed by two customers.
(iii) Every customer can borrow only one loan from the same branch.
(iv) No two loans from the same branch can have the same amount.

Answer.

(i) $cid \rightarrow lid$
(ii) $lid \rightarrow cid$
(iii) $cid, branch \rightarrow lid$
(iv) $bid, amount \rightarrow lid$

Problem 4. Consider the table in the previous problem. Suppose that we would like to enforce the functional dependency: $bid, amount \rightarrow lid$. Write an SQL statement to check whether the functional dependency.

Answer.

```
select bid, amount from LOAN
group by bid, amount
having count (distinct lid) > 1
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There is violation if and only if the query returns a non-empty result.