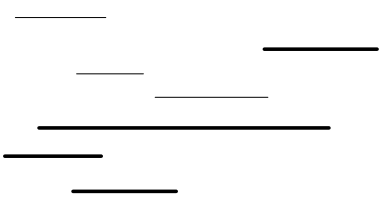
CSC6210 (Fall 2009) Assignment 2 (Due 20 Dec)

Problem 1 (40%).
Give an algorithm to build an external priority search tree on a set of N points in $O(\frac{N}{B}\log_B N)$ I/Os, assuming that the memory size M at least B^2.

Note: the external priority search tree is the structure discussed in the lecture that can answer 3-sided range search optimally. See Section 7 of Lars' lecture notes.

Problem 2 (60%).
The *lower envelop* of a set S of horizontal segments contains all such segments $s \in S$ that there is a vertical line ℓ such that s is the lowest segment in S intersecting ℓ. For instance, assume that S has 7 segments as shown below. Then the lower envelop consists of the 4 bold segments.



Let N be the number of segments in S. Give an algorithm that finds the lower envelop in $O(\frac{N}{B}\log_B N)$ I/Os, assuming that the memory size M is at least B^2.

Bonus problem.
Consider a structure that can be built in $O(\frac{N}{B}\log_B N)$ I/Os (where N is the number of items indexed), and answer a query in $O(\log_B N)$ I/Os. We already know that, using logarithmic rebuilding, we can make the structure semi-dynamic by supporting insertions in $O(\log_B^2 N)$ amortized I/Os and queries in $O(\log_B^2 N)$ I/Os.

Improve logarithmic rebuilding to obtain a semi-dynamic structure that supports queries still in $O(\log_B^2 N)$ I/Os, but insertions in $O(\frac{1}{\sqrt{B}}\log_B^2 N)$ I/Os.

(Hint: cleverly adjust the sizes of the $O(\log_B N)$ structures in logarithmic rebuilding.)