CSCI5020 External Memory Data Structures: Exercise List 4

In the following problems, $B$ is the block size, and $M$ is the memory capacity.

**Problem 1.** Assuming $M \geq B^2$, describe an algorithm to construct an external interval tree on $N$ intervals in $O\left(\frac{N}{B} \log_B N\right)$ I/Os.

**Problem 2 (Ray Shooting on Rays).** Let $S$ be a set of $N$ horizontal rays in $\mathbb{R}^2$ shooting towards right, i.e., each ray in $S$ has the form $[x, \infty) \times y$. Given a point $q$ in $\mathbb{R}^2$, a ray shooting query finds the first ray that is hit by a vertical ray shooting upwards from $q$. Describe a structure that uses $O(N/B)$ space and answers a ray shooting query in $O(\log_B N)$ I/Os. Make your structure fully dynamic such that each insertion and deletion can be supported in $O(\log_B N)$ I/Os.

**Problem 3.** Let $L = \{\ell_1, \ldots, \ell_l\}$ be a set of $l$ vertical lines in $\mathbb{R}^2$, where $l = \sqrt{B}$. Let $S$ be a set of $N$ horizontal segments such that each segment in $S$ has its endpoints on two different lines in $L$. Given a vertical ray $r$ shooting downwards from a point, a query reports all the segments in $S$ intersecting $r$. Give a structure on $S$ that consumes $O(N/B)$ space, and answers a query in $O(1 + K/B)$ I/Os, where $K$ is the number of segments reported. Your structure also needs to support an insertion and deletion in $O(\log_B N)$ I/Os amortized, assuming $M \geq B^2$.

**Problem 4 (Ray Intersecting Segments).** Let $S$ be a set of $N$ horizontal segments in $\mathbb{R}^2$. Given a vertical ray $r$ shooting downwards from a point, a query reports all the segments in $S$ intersecting $r$. Describe a structure on $S$ that consumes $O(N/B)$ space, and answers a query in $O(\log_B N + K/B)$ I/Os, where $K$ is the number of segments reported. Your structure also needs to support an insertion and a deletion in $O(\log_B N)$ I/Os amortized, assuming $M \geq B^2$. 