CSCI 5020 External Memory Data Structures: Exercise List 3

In the following problems, $B$ is the block size, and $M$ is the memory capacity.

Problem 1 (Interval Stabbing). Let $S$ be a set of $n$ intervals in $\mathbb{R}$. Given a real value $q$, a stabbing query finds all the intervals in $S$ that contain $q$. Describe a data structure of $O(n/B)$ space that answers such a query in $O(\log B n + k/B)$ I/Os where $k$ is the number of intervals reported. For example, if $I = \{[2, 9], [3, 12], [5, 7], [8, 18]\}$, then a query with $q = 6$ returns the first 4 intervals in $I$.

Problem 2 (Segment Intersection). Let $S$ be a set of $n$ horizontal segments in $\mathbb{R}^2$ (namely, each segment is in the form $[x_1, x_2] \times y$). Given a vertical segment $q = x \times [y_1, y_2]$, a query reports all the segments in $S$ intersecting $q$. Describe a data structure of $O(n/B)$ space that answers such a query in $O(\log B n + k/B)$ I/Os where $k$ is the number of segments reported.

Problem 3 (1D Range Sum). Let $P$ be a set of $n$ points in $\mathbb{R}$. Each point in $P$ is associated with a real-valued weight. Given an interval $q = [x, y]$ in $\mathbb{R}$, a range sum query returns the sum of all the weights in $P \cap q$. Design a data structure of $O(n/B)$ space that answers a query in $O(\log B n)$ I/Os, and supports an insertion and a deletion in $O(\log B n)$ I/Os.

Problem* 4 (Segment Intersection Sum). Let $S$ be a set of $n$ horizontal segments in $\mathbb{R}^2$, each of which is associated with a real-valued weight. Given a vertical segment $q = x \times [y_1, y_2]$, a query reports the sum of all the segments in $S$ intersecting $q$. Describe a data structure of $O(\frac{n}{B} \log B n)$ space that answers such a query in $O(\log B n)$ I/Os.