CSCI 5020 External Data Structures: Exercise List 4

In the following problems, $B$ is the block size, and $M$ is the memory capacity.

**Problem 1.** Modify the external interval tree to achieve the same performance (i.e., linear space, logarithmic query time, and logarithmic update time amortized) under the assumption that $M = B^{1.5}$.

**Problem 2.** Assuming $M \geq B^2$, describe an algorithm to construct an external interval tree on $N$ intervals in $O(\frac{N}{B} \log_B N)$ I/Os.

**Problem 3 (Ray shooting on rays).** Let $S$ be a set of $N$ horizontal rays in $\mathbb{R}^2$ shooting towards right, i.e., each ray in $S$ has the form $[x, \infty) \times y$. Given a point $q$ in $\mathbb{R}^2$, a ray shooting query finds the first ray that is hit by a vertical ray shooting upwards from $q$. Describe a structure that uses $O(N/B)$ space and answers a ray shooting query in $O(\log_B N)$ I/Os. Make your structure fully dynamic such that each insertion and deletion can be supported in $O(\log_B N)$ I/Os.