# Exercises for CSCI 5010

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**Problem 1.** Let $H$ be a set of halfplanes in $\mathbb{R}^2$ in general position (i.e., no two halfplanes' boundary lines are parallel). Prove: $H$ is infeasible for LP if and only if $H$ has 3 halfplanes with an empty intersection.

**Problem 2 (Reading Exercise).** Extend our 2D LP algorithm to handle also the case where the input set $H$ of halfplanes is infeasible.

Hint: This is not too hard, but if you do not want to think, read Pg 51 of Prof. Mount's notes.

**Problem 3.** In the lecture, we proved that our 2D LP algorithm runs in $O(n)$ expected time when the input set $H$ of halfplanes is feasible. Extend the proof to the case where $H$ is infeasible. You can still make the general position assumption that no three planes' boundary lines pass the same point.

Hint: Use the result of Problem 1 in backward analysis.

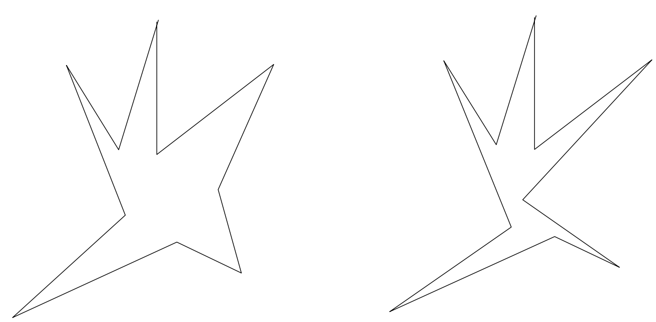
**Problem 4.** Prove: our 2D LP algorithm runs in $O(n)$ expected time even if the boundary lines of three (or more) halfplanes pass the same point. You can focus on the case where the problem is feasible.

Hint: Let us consider the processing of the last plane in $H$. We proved that we had to spend $O(n)$ time on this plane with a probability at most $2/n$. Show that this is the case no matter how many planes' boundary planes pass the same point.

**Problem 5 (Reading Exercise).** Give an algorithm to solve LP in $O(n)$ expected time in 3D space.

Hint: This is not too hard, but if you do not want to think, read Pg 51 of the lecture notes or Chapter 4 of the textbook.

**Problem* 6 (textbook exercise 4.15).** A polygon is *star-shaped* if there is a point $p$ inside the polygon that is visible to all the vertices of the polygon (two points in a polygon are visible to each other if the segment connecting the two points is completely inside the polygon). In the figure below, the left polygon is star-shaped but the right one is not. Given a polygon of $n$ vertices, determine in $O(n)$ expected time whether it is star-shaped.



Hint: Cast this into an LP problem.