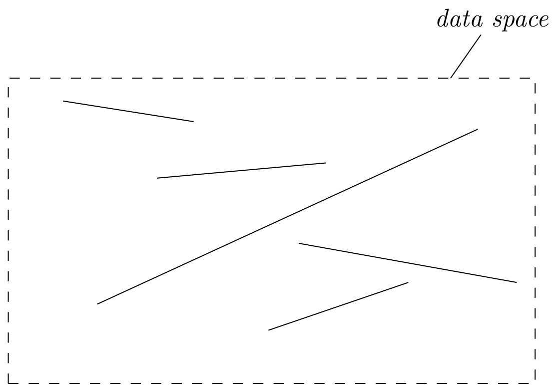
CSCI5010: Final Exam

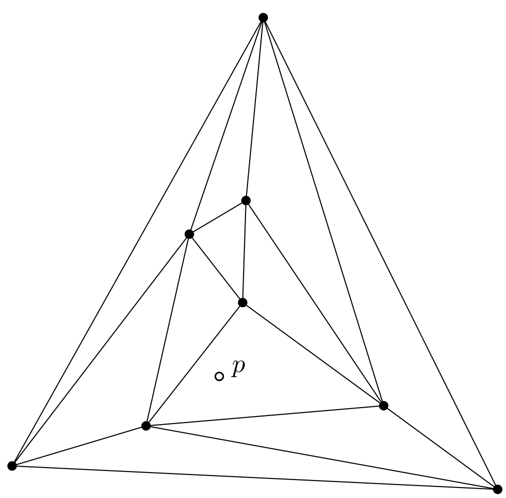
Name: **Student ID:**

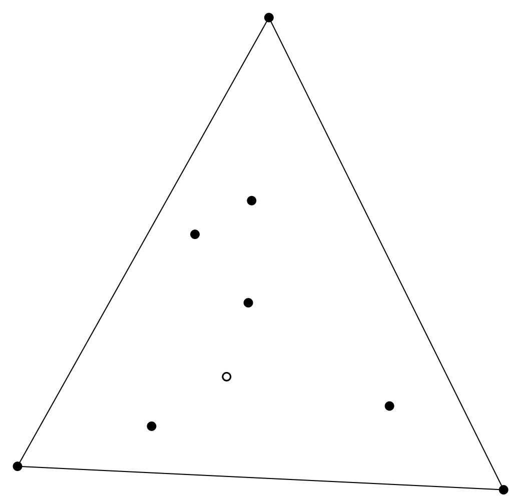
Please write all your solutions in the answer book, except Problems 1 and 2 which should be answered in this paper.

Problem 1 (10%). The figure below shows a set of 5 segments. Give the trapezoidal map that is decided by these segments.



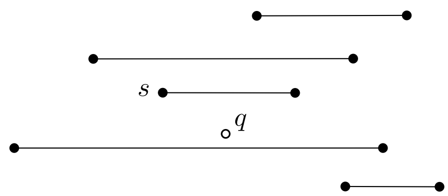
Problem 2 (10%). The left figure below shows the Delaunay triangulation of the set of black points. Suppose that we want to insert point p (i.e., the white point). Draw the resulting Delaunay triangulation in the figure on the right.



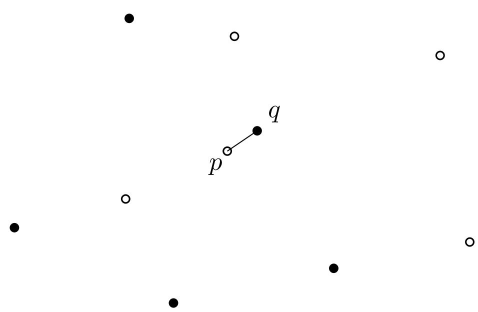


Problem 3 (20%). Let P be a set of n points in $\mathbb{R}^2$. Given an axis-parallel rectangle q, a query reports the number of points in $q \cap P$. Describe a data structure of $O(n)$ size that answers such a query in $O(\sqrt{n})$ time.

Problem 4 (20%). Let S be a set of horizontal segments in $\mathbb{R}^2$, where each segment has the form $[x_1, x_2] \times y$. Given a point q, a query reports the first segment of S that will be hit if we shoot a ray upwards from q (e.g., in the figure below, the query reports s). Preprocess S into a data structure of $O(n)$ space such that a query can be answered in $O(\log n)$ time.



Problem 5 (20%). Let S and T be two sets of points in $\mathbb{R}^2$. Let (p, q) be a *closest pair* of S and T, namely, the Euclidean distance between p and q is the smallest among all pairs of points in $S \times T$. For example, in the figure below, let S (T) be the set of black (white) points. The closest pair is the two points between which there is a segment. Prove that there must be an edge between p and q in the Delaunay triangulation of $S \cup T$.



Problem 6 (20%). Let P be a set of n points in $\mathbb{R}^2$. Given a rectangle r and a query point q, a *constrained nearest neighbor* query returns the point in $P \cap r$ that has the smallest Euclidean distance to q (i.e., among all the points of P falling in r, report the one closest to q). For example, in the figure below, let P be the set of black points; given the rectangle r and q as shown, a query returns point p_1 as its answer (note that the answer is not p_2 as it is outside r). Give a structure of $O(n \log^2 n)$ space that answers such a query in $O(\log^3 n)$ time.