

## CSCI5010 Exercise List 7

**Problem 1 (Smallest Enclosing Square: Semi-Uniqueness).** Let  $P$  be a set of  $n$  points in  $\mathbb{R}^2$ . An axis-parallel square  $S$  is an *enclosing square* of  $P$  if all the points of  $P$  are covered by  $S$ . Prove that

- There can be two enclosing squares of  $P$  with the minimum side length.
- If there are two enclosing squares of  $P$  with the minimum side length, then their centers must either have the same x-coordinate, or the same y-coordinate.

**Problem 2 (Smallest Enclosing Square: Algorithm).** Design an  $O(n)$  expected time algorithm to compute the minimum enclosing square of  $P$ .

**Problem 3 (Range Count).** Let  $P$  be a set of  $n$  points in  $\mathbb{R}^2$ . Given an axis-parallel rectangle  $q$ , a *range count* query returns the number of points of  $P$  that are covered by  $q$ . Design a structure of  $O(n \log n)$  space that answers a query in  $O(\log^2 n)$  time.

**Problem 4 (Range Count).** Improve the query time of your structure for Problem 3 to  $O(\log n)$  using fractional cascading. The space of your structure should remain  $O(n \log n)$ .