CSCI5010 Exercise List 12

Problem 1 (Gabriel Graph; Problem 9.13 of the Textbook). Let P be a set of points in $\mathbb{R}^2$. P defines a *gabriel graph* G as follows. Each vertex of G corresponds to a distinct point of P. Two vertices p_1, p_2 are connected by an edge in G if and only if the circle with segment p_1p_2 as a diameter does not cover any point of P in its interior. Prove:

- If there is an edge between p_1 and p_2 in G, then there is a Delaunay edge between p_1 and p_2.
- There is an edge between p_1 and p_2 in G if and only if the Delaunay edge between p_1 and p_2 intersects the boundary edge of the Voronoi cells of p_1 and p_2.

Problem 2 (k-Clustering; Problem 9.16 of the Textbook). A *k-clustering* of a set P of n points in $\mathbb{R}^2$ is a partitioning of P into k non-empty subsets $P_1, ..., P_k$. Define the distance between any pair P_i, P_j of clusters to be the minimum distance between one point from P_i and one point from P_j, namely:

$$dist(P_i, P_j) \quad = \quad \min_{p \in P_i, q \in P_j} dist(p, q)$$

We want to find a k-clustering that maximizes the minimum distance between clusters, namely, to maximize $\min_{i \neq j} dist(P_i, P_j)$.

- Suppose that $\min_{i \neq j} dist(P_i, P_j)$ is achieved by points $p \in P_i, q \in P_j$. Prove that the segment pq is an edge of the Delaunay triangulation of P.
- Give an $O(n \log n)$ time algorithm to compute such a k-clustering.