## CSCI3610: Special Exercise Set 3

Problem 1. If we run the activity-selection algorithm taught in the class on the following input: $S=\{[1,10],[2,22],[3,23],[20,30],[25,45],[40,50],[47,62],[48,63],[60,70]\}$ what is the set of intervals returned?

Problem 2. The following is another greedy algorithm for the activity selection problem. Initialize an empty $T$, and then repeat the following steps until $S$ is empty:

- (Step 1) Add to $T$ the interval $I$ with the shortest length.
- (Step 2) Remove from $S$ the interval $I$, and all the intervals overlapping with $I$.

Finally, return $T$ as the answer.
Prove: the above algorithm does not always return an optimal solution.
Problem 3 (Fractional Knapsack). Let $\left(w_{1}, v_{1}\right),\left(w_{2}, v_{2}\right), \ldots,\left(w_{n}, v_{n}\right)$ be $n$ pairs of positive real values. Given a real value $W \leq \sum_{i=1}^{n} w_{i}$, we want to find $x_{1}, x_{2}, \ldots, x_{n}$ to maximize the objective function

$$
\sum_{i=1} \frac{x_{i}}{w_{i}} \cdot v_{i}
$$

subject to

- $0 \leq x_{i} \leq w_{i}$ for every $i \in[1, n]$;
- $\sum_{i=1}^{n} x_{i} \leq W$.
W.l.o.g., assume that $v_{1} \geq v_{2} \geq \ldots \geq v_{n}$. Consider the algorithm that works as follows.

1. for $i \leftarrow 1$ to $n$ do
2. $\quad x_{i} \leftarrow \min \left\{W, w_{i}\right\}$
3. $W \leftarrow W-x_{i}$

Prove: the above algorithm does not always returns an optimal solution.
Problem 4 (0-1 Knapsack). Suppose that there are $n$ gold bricks, where the $i$-th piece weighs $p_{i}$ bounds and is worth $d_{i}$ dollars. Given a positive integer $W$, our goal is to find a set $S$ of gold bricks such that

- the total weight of the bricks in $S$ is at most $W$, and
- the total value of the bricks in $S$ is maximized (among all the sets $S$ satisfying the first condition).

Assuming $d_{1} \geq d_{2} \geq \ldots \geq d_{n}$, let us consider the following greedy algorithm:

1. $S=\emptyset$
2. for $i=1$ to $n$
3. if $p_{i} \leq W$ then
4. add $p_{i}$ to $S ; W \leftarrow W-p_{i}$

Prove: the above algorithm does not guarantee finding the desired set $S$.

