CSCI3610: Special Exercise Set 1

Problem 1. Explain how to implement the operation $x \mod y$ in $O(1)$ time where x and y are positive integers.

Problem 2. For the k-selection problem, suppose that the input is an array of 12 elements: (58, 23, 98, 83, 32, 24, 18, 45, 85, 91, 2, 34). Recall that our randomized algorithm first selects a number v and then recursively solves a subproblem. Suppose that $v = 34$ and $k = 10$. What is the size of the array for the subproblem?

Problem 3 (Textbook Exercise 9.3-5). The *median* of a set S of n elements is the $\lfloor n/2 \rfloor$ smallest element in S. Suppose that you are given a deterministic algorithm for finding the median of S (stored in an array) in $O(n)$ worst-case time. Using this algorithm as a black box, design another deterministic algorithm for solving the k-selection problem (for any $k \in [1, n]$) in $O(n)$ worst-case time.

Problem 4. Let S be a set of n distinct integers, and k_1, k_2 be arbitrary integers satisfying $1 \leq k_1 \leq k_2 \leq n$. Suppose that S is given in an array. Give an $O(n)$ expected time algorithm to report *all* the integers whose ranks in S are in the range $[k_1, k_2]$. Recall that the rank of an integer v in S equals the number of integers in S that are at most v.

Problem 5* (Textbook Exercise 9-2). We are given an array that stores a set S of n distinct positive integers. Set $W = \sum_{e \in S} e$. Describe an algorithm to find the element $e^* \in S$ that makes both of the following hold:

- $\sum_{e < e^*} e < W/2$
- $\sum_{e > e^*} e \leq W/2$.

Your algorithm should finish in $O(n)$ time ($O(n)$ expected time is acceptable).
(Hint: First convince yourself that such e^* is unique, and then adapt the k-selection algorithm).