# Dynamic Programming 2: Rod Cutting

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## The Rod Cutting Problem

**Input:**

- a rod of length $n$
- an array $P$ of length $n$ where
  $P[i]$ is the price for a rod of length $i$, for each $i \in [1, n]$

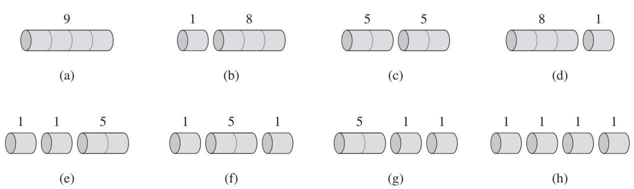
**Goal:** Cut the rod into segments of integer lengths to maximize the revenue.

## Example

Price array $P$

| length $i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| price $P[i]$ | 1 | 5 | 8 | 9 |

All possible ways to cut a rod of length 4:




(by courtesy of the textbook)

The optimal cutting method: (c), which has a revenue of 10

The key to solving the problem is to identify its underlying **recursive structure**.

Specifically, how the original problem is related to subproblems.

The recursive structure will then point to an algorithm based on dynamic programming.

Define $opt(n)$ as the optimal revenue from cutting up a rod of length $n$.

Clearly, $opt(0) = 0$.

Consider now $n \geq 1$.
Let $i$ be the length of the first segment.

- $i$ can be any integer in $[1, x]$.

**Conditioned on** the first segment having length $i$, the highest revenue attainable is $P[i] + opt(n - i)$.

Therefore:

$$opt(n) = \max_{i=1}^{n} (P[i] + opt(n - i))$$

We have obtained a recursive structure for the problem.

Given

$$opt(n) = \max_{i=1}^{n} (P[i] + opt(n-i))$$

we can compute $opt(n)$ in $O(n^2)$ time using dynamic programming (this is the problem solved in the last lecture).

Wait! We need to **generate** a cutting method to achieve revenue $opt(n)$.

This can be done by recording which subproblem yields $opt(n)$.

See the next slide.

Given

$$opt(n) = \max_{i=1}^{n}(P[i] + opt(n-i))$$

define $bestSub(n) = k$ if maximization is obtained at $k = i$ (i.e., first segment having length $k$).

**Example**

| length $i$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| price $P[i]$ | 1 | 5 | 8 | 9 |

$bestSub(4) = 2$, $bestSub(3) = 3$, $bestSub(2) = 2$, $bestSub(1) = 1$

After we have computed $bestSub(i)$ for every $i \in [1, n]$, the best method for cutting up a rod of length $n$ can be obtained in $O(n)$ time.

(Think: why?)

For each $i \in [1, n]$, computing *bestSub*$(i)$ is no more expensive than computing $opt(i)$. This is left as a regular exercise.

We conclude that the rod cutting problem can be solved in $O(n^2)$ time.