Problem 1. Let $S$ be a set of $n$ intervals $\{[s_i, f_i] \mid 1 \leq i \leq n\}$, satisfying $f_1 \leq f_2 \leq \ldots \leq f_n$. Denote by $S'$ the set of intervals in $S$ that are disjoint with $[s_1, f_1]$. Prove: if $T' \subseteq S'$ is an optimal solution to the activity selection problem on $S'$, then $T' \cup \{[s_1, f_1]\}$ is an optimal solution to the activity selection problem on $S$.

(Note: This completes the induction step of the correctness proof discussed in the class.)

Problem 2. Describe how to implement the activity selection algorithm discussed in the lecture in $O(n \log n)$ time, where $n$ is the number of input intervals.

Problem 3. Prof. Goofy proposes the following greedy algorithm to “solve” the activity selection problem. Let $S$ be the input set of intervals. Initialize an empty $T$, and then repeat the following steps until $S$ is empty:

- (Step 1) Add to $T$ the interval $I = [s, f]$ in $S$ that has the smallest $s$-value.
- (Step 2) Remove from $S$ (i) the interval $I$, and (ii) all the intervals that overlap with $I$.

Finally, return $T$ as the answer.

Prove: the above algorithm does not guarantee an optimal solution.

Problem 4**. Prof. Goofy is giving another try! This time he proposes a more sophisticated greedy algorithm. Again, let $S$ be the input set of intervals. Initialize an empty $T$, and then repeat the following steps until $S$ is empty:

- (Step 1) Add to $T$ the interval $I \in S$ that overlaps with the fewest other intervals in $S$.
- (Step 2) Remove from $S$ the interval $I$ as well as all the intervals that overlap with $I$.

Finally, return $T$ as the answer.

Prove: the above algorithm does not guarantee an optimal solution.