CSCI: Regular Exercise Set 2

Prepared by Yufei Tao

Problem 1. Prove $30\sqrt{n} = O(\sqrt{n})$.

Problem 2. Prove $\sqrt{n} = O(n)$.

Problem 3. Let $f(n)$, $g(n)$, and $h(n)$ be functions of integer n. Prove: if $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.

Problem 4. Prove $(2n+2)^3 = O(n^3)$.

Problem 5. Prove or disprove: $4^n = O(2^n)$.

Problem 6. Prove or disprove: $\frac{1}{n} = O(1)$.

Problem 7*. Prove that if $k \log_2 k = \Theta(n)$, then $k = \Theta(n/\log n)$.

Problem 8. We can extend the big-O notation to multiple variables. In this problem, we will focus on two variables, but the idea extends to more variables in a straightforward manner.

Formally, let $f(n,m)$ and $g(n,m)$ be functions of variables n and m satisfying $f(n,m) \geq 0$ and $g(n,m) \geq 0$. We say $f(n,m) = O(g(n,m))$ if there exist constants c_1 and c_2 such that $f(n,m) \leq c_1 \cdot g(n,m)$ holds for all $n \geq c_2$ and $m \geq c_2$.

Prove:

- $n^2m + 100nm = O(n^2m)$.
- $n^2m + 100nm^2 = O(n^2m + nm^2)$.