CSCI2100: Regular Exercise Set 13

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Problem 1. Let S be a set of integer pairs of the form (id, v). We will refer to the first field as the *id* of the pair, and the second as the *key* of the pair. Design a data structure that supports the following operations:

- Insert: add a new pair (id, v) to S (you can assume that S does not already have a pair with the same id).

- Delete: given an integer t, delete the pair (id, v) from S where $t = id$, if such a pair exists.

- DeleteMin: remove from S the pair with the smallest key, and return it. .

Your structure must consume $O(n)$ space, and support all operations in $O(\log n)$ time where $n = |S|$.

Problem 2. Describe how to implement the Dijkstra's algorithm on a graph $G = (V, E)$ in $O((|V| + |E|) \cdot \log |V|)$ time.

Problem 3*. In the lecture, we proved the correctness of Dijkstra's algorithm in the scenario where all the edges have positive weights. Prove: the algorithm is still correct if we allow edges to take *non-negative* weights (i.e., zero weights are allowed).

Problem 4. Consider again your proof for Problem 1. Point out the place that requires edge weights to be non-negative.