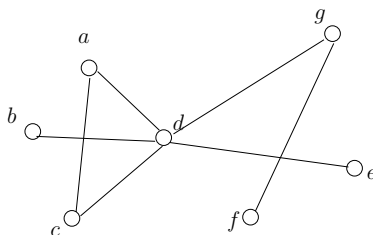


CSCI2100: Regular Exercise Set 11

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Problem 1 (DFS on Undirected Graphs). Let $G = (V, E)$ be an undirected graph. Consider the execution of DFS on G . The algorithm runs in exactly the same way as DFS on a directed graph. The only difference is that, a vertex u is popped out of the stack, only if none of its neighbors (instead of out-neighbors) is still white. Give a possible DFS tree produced if we (i) start DFS on a in the following graph, and (ii) follow the convention that we explore the neighbors of a vertex in alphabetic order.



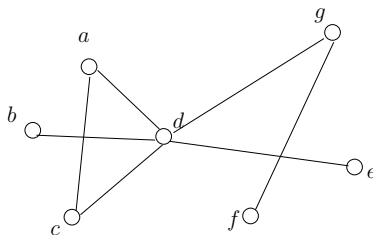
Problem 2 (No Cross Edges in Undirected DFS). Let $G = (V, E)$ be an undirected graph. Consider the DFS forest produced by running DFS on G (assuming arbitrary starting and re-starting vertices). Let $\{u, v\}$ be an edge in G (note that we use the notation $\{u, v\}$, instead of (u, v) , to emphasize that the edge has no directions). Prove: either u is an ancestor of v , or v is an ancestor of u .

Remark: Because of this lemma, we can classify each edge $\{u, v\}$ in G as follows:

- *Tree edge:* if u is the parent of v or v is the parent of u .
- *Back edge:* otherwise.

Problem 3 (Undirected Cycle Detection). Let $G = (V, E)$ be an undirected graph. A *cycle* is a sequence of edges $\{v_1, v_2\}, \{v_2, v_3\}, \dots, \{v_{t-1}, v_t\}$ where $v_t = v_1$. Adapt DFS to design an algorithm to detect whether G has a cycle in $O(|V| + |E|)$ time.

Problem 4 (Articulation Vertex).** Let $G = (V, E)$ be an undirected graph that is connected (i.e., there is a path between any two distinct vertices). A vertex $u \in V$ is called an *articulation vertex* if the following is true: G becomes disconnected after removing u and all the edges of u . For example, in the figure below, vertex g is an articulation, and so is d . No other vertices are articulation vertices.



Consider any DFS tree on G . Prove:

- If a vertex u is a leaf in the DFS tree, it cannot be an articulation vertex.
- Let u a vertex that is neither a leaf in the DFS tree nor the root. It is an articulation vertex if and only if the following is true:
 - There is at least one child v of u , such that no back edge connects a descendant of v to a proper ancestor of u .
- Let u be the root of a DFS tree. It is an articulation vertex if and only if it has at least two child nodes in the DFS tree.

Problem 5* (Finding an Articulation Vertex). Let $G = (V, E)$ be an undirected graph that is connected. Design an algorithm to determine whether G has any articulation vertex. Your algorithm must finish in $O(|V| + |E|)$ time.