Problem 1. Consider the weighted directed graph below.

Suppose that we run Dijkstra’s algorithm starting from vertex $a$. Recall that the algorithm relaxes the outgoing edges of every other vertex in turn. Give the order of vertices by which the algorithm relaxes their edges.

Problem 2. Let $G = (V, E)$ be a weighted directed graph. Give an algorithm to compute the shortest path distances between all pairs of vertices. Your algorithm should finish in $O(|V|(|V| + |E|) \log |V|)$ time.

Problem 3. Adapt Dijkstra’s algorithm to solve the SSSP problem on a weighted undirected graph.

Problem 4. Consider the weighted undirected graph below.

Suppose that we run Prim’s algorithm to find a minimum spanning tree (MST) of this graph. Explain in what order does the algorithm insert edges into the tree.

Problem 5. Consider again the execution of Prim’s algorithm in Problem 4. Indicate how $\text{best-ext}(c)$ changes during the execution (i.e., what is $\text{best-ext}(c)$ after the first edge is included in the MST, what is it after the second edge has been included, and so on).