Problem 1. Prove: A tree with \( n \) nodes has \( n - 1 \) edges.

Problem 2 (Max Heap). The binary heap we discussed in the class is called the min-heap because of the delete-min operation. Conversely, a max-heap on a set \( S \) of integers aims to support insertions and the following delete-max operation:

- **Delete-max**: Reports the largest integer in \( S \), and removes it from \( S \).

Describe how a min-heap can be used to implement a max-heap without changing its structure and algorithms. Your max-heap must still use \( O(|S|) \) space, and support an insertion and a delete-max operation in \( O(\log |S|) \) time.

Problem 3* (Priority Queue with Attrition). Let \( S \) be a dynamic set of integers. At the beginning \( S \) is empty. We want to support the following operations:

- **Insert-with-Attrition**(e): First removes all integers in \( S \) that are greater than \( e \), and then adds \( e \) to \( S \).
- **Delete-Min**: Removes and returns the smallest integer of \( S \).

For example, suppose we perform the following sequence of operations:

1. Insert-with-Attrition(83)
2. Insert-with-Attrition(5)
3. Insert-with-Attrition(10)
4. Insert-with-Attrition(15)
5. Insert-with-Attrition(12)
6. Delete-Min
7. Delete-Min

After Operation 3, \( S = \{5, 10\} \) (note that 83 has been deleted by Operation 2). After Operation 5, \( S = \{5, 10, 12\} \). After Operation 6, \( S = \{10, 12\} \).

Describe a data structure with the following guarantees:

- At all times, the space consumption is \( O(|S|) \).
- Any sequence of \( n \) operations (each being an insert-with-attrition or delete-min) is processed with \( O(n) \) time, i.e., \( O(1) \) amortized time per operation.

Problem 4 (Textbook Exercise 6.5-9). Suppose that we have \( k \) arrays \( A_1, A_2, ..., A_k \) of integers, such that each array has been sorted in ascending order. Let \( n \) be the total number of integers in those arrays. Describe an algorithm to produce an array that sorts all the \( n \) integers in ascending order (you may assume that no integer exists in two arrays). Your algorithm must finish in \( O(n \log k) \) time.

For example, suppose that \( k = 3 \), and that the three arrays are \( (2, 23, 32, 35, 37) \), \( (5, 10) \), and \( (33, 58, 82) \). Then you should produce an array containing \( (2, 5, 10, 23, 32, 33, 35, 37, 58, 82) \).