Problem 1. After applying the following operations to an empty stack:

push(35), push(36), push(43), pop, pop, push(8), push(51), pop

What is the content of the stack? Answer: [    ]

A. 8, 51  
B. 36, 43  
C. 35, 8   
D. 35, 51

Answer: C

Problem 2. After applying the following operations to an empty queue:

enqueue(35), enqueue(36), enqueue(43), dequeue, dequeue, enqueue(8), enqueue(51), dequeue

What is the content of the queue? Answer: [    ]

A. 8, 51  
B. 36, 43  
C. 35, 8   
D. 35, 51

Answer: A

Problem 3. Which of the following operations does not have $O(1)$ cost? Answer: [    ]

A. Pushing an element into a stack.  
B. Inserting an element into a linked list.  
C. Dequeuing an element from a queue.  
D. Deleting an element 10 from a linked list without knowing which node contains 10.

Answer: D

Problem 4. Which of the following are true? There is more than one correct choice; no marks are given unless you identify all the correct ones. Answer: [    ]

A. Consider a stack implementation that supports push and pop each in $O(1)$ amortized time. Thus, any sequence of $t$ pushes followed by $t/2$ pops can be processed in $O(t)$ worst case time, regardless of the value of $t$.
B. Our dynamic-array implementation of the stack sometimes needs $O(n)$ time to process a push, when there are $n$ elements in the stack.
C. By choosing a hash function uniformly at random from a universal hash family, we guarantee $O(1)$ amortized lookup cost in dictionary search.

Answer: AB
Problem 5. Consider $S = \{1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15\}$. We build a hash table on $S$ with hash function $h(k) = 1 + k \mod 4$. If we look up an element $q$ in the hash table, which of the following value of $q$ would have the lowest look up cost? Answer: [  ]
A. 21   B. 22   C. 23   D. 24

Answer: B

Problem 6. Consider the hash table in Problem 5. Show the content of the linked list for each possible hash key.

Answer: Linked list for hash value 1: (4, 8, 12).
For hash value 2: (1, 5, 9, 13).
For hash value 3: (10).
For hash value 4: (3, 7, 11, 15).

Problem 7. Let $S_1$ and $S_2$ be two sets of integers, such that $|S_1| = |S_2| = n$. Give an algorithm to report all the integers in $S_1 \cap S_2$ in $O(n)$ expected time.

Answer: Create a hash table on $S_1$ in $O(n)$ time. For each element $e \in S_2$, probe the hash table to see if $e \in S_1$. 