Exercises: Vector Derivative

**Problem 1.** Solve the following limits:

1. \( \lim_{t \to 3} f(t) \), where \( f(t) = [5t + 3, \frac{\sin(t-3)}{t-3}] \).
2. \( \lim_{t \to 0} f(t) \), where \( f(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}] \).
3. \( \lim_{t \to 0} f(t) \), where
   \[
   f(t) = \begin{cases} 
   [5t^2 + 3t, t^2, \frac{e^t-1}{t}] & \text{if } t \neq 0 \\
   [10, 10, 10] & \text{otherwise}
   \end{cases}
   \]

**Problem 2.** Discuss the continuity of \( f(t) \) at \( t = 0 \).

1. \( f(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}] \).
2. \( f(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}] \) if \( t \neq 0 \) otherwise, \( f(t) = [10, 10, 10] \).
3. \( f(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}] \) if \( t \neq 0 \) otherwise, \( f(t) = [0, 0, 1] \).

**Problem 4.** Suppose that \( f(t) = [\sin(t), \cos(t^3), 5t^2] \). Answer the following questions:

1. Give the function \( f'(t) \).
2. Give the function \( f''(t) \) (which is the derivative of \( f'(t) \)).
3. Give the function \( f'''(1) \) (where \( f'''(t) \) is the derivative of \( f''(t) \)).

**Problem 5.** Suppose that \( f(t) = [t^2, \sin(t), 2t] \) and \( g(t) = 2ti + \frac{1}{\sin(t)}j + 3t^2k \).

1. Give the function \( h(t) = f(t) \cdot g(t) \).
2. Give the function \( h'(t) \).
3. Give the function \( f'(t) \) and \( g'(t) \).
4. Verify that \( h'(t) = f'(t) \cdot g(t) + g'(t) \cdot f(t) \).

**Problem 6.** Suppose that \( f(t) = [t, t^2, 1] \) and \( g(t) = [1, t, t^2] \).

1. Give the function \( h(t) = f(t) \times g(t) \).
2. Give the function \( h'(t) \).
3. Verify that \( h'(t) = f'(t) \times g(t) + f(t) \times g'(t) \).