## Homework 2

Due at 5pm, Oct 25. Give your answer sheet to TA (Liu Yang, yliu@cse).

1. Consider two sets of quantum pure states $\left\{\left|u\_{i}\right〉:i=1, …, n\right\}$ and $\left\{\left|v\_{i}\right〉:i=1, …, n\right\}$. Prove that there is a unitary operation $U$ s.t. $U\left|u\_{i}\right〉=\left|v\_{i}\right〉$ for all $i\in \left[n\right]$ if and only if $\left〈u\_{j}\right〉=\left〈v\_{j}\right〉$ for all $i,j\in [n]$..
2. Prove that for any two matrices $A,B\in C^{n×n}$, $\left〈A,B\right〉\leq \left‖A\right‖⋅\left‖B\right‖\_{tr}$. Recall that $\left〈A,B\right〉≝tr\left(A^{†}B\right)$, the operator norm $\left‖A\right‖$ is the largest singular value of $A$, and the trace norm $\left‖B\right‖\_{tr}$ is the summation of all singular values of $B$.
3. What is the bounded-error quantum query complexity of the function of AND-of-OR? Here the function is defined as follows. It has $n=kl$ variables, divided into $k$ blocks $x^{1}, …, x^{k}$, with $l$ variables $x\_{1}^{i}, …, x\_{l}^{i}$ in each block $x^{i}$. The function $f\left(x\right)=1$ if for all $i\in [k]$, $x^{i}$ contains at least one variable $x\_{1}^{i}, …, x\_{l}^{i}$ being 1. (Otherwise $f\left(x\right)=0$.) In other words, the function is the AND of the $k$ blocks, and each block is the OR function on its $l$ variables.
Note that to get the quantum query complexity, you need to prove both the upper and lower bounds, and hopefully they match (up to a constant factor).