How Can Computer Science Help When Your Drinking Water Gets too Salty?

Jimmy Lee
Department of Computer Science and Engineering
The Chinese University of Hong Kong

In collaboration with the International Institute for Software Technology
United Nations University

Controlling Salinity in a Potable Water Supply System Using a Constraint Programming Approach

Jimmy Lee
Department of Computer Science and Engineering
The Chinese University of Hong Kong

In collaboration with the International Institute for Software Technology
United Nations University
Outline

• Domain Description

• Constraint Programming (CP)

• Problem Modelling

• Improvements

• Concluding Remarks

MSC Seminar (January 30, 2012)
Increasing Salinity in Water

- **Salinity** is the concentration of salts in water
- Decrease of river flow during **dry** seasons
- **Intrusion** of sea water
- Duration depends on unforeseen factors such as tidal flow and weather conditions

The Raw Water System
The Salinity Problem

How Serious is the Problem?

- Year 2004 was one of the driest years in the past 50 years and the situation has got only worse

- Rainfall reduced by more than 30% of average

- Salinity of raw water can drastically rise to such levels as 2500 ppm (c.f. ≤ 250 ppm ideally)

- Affecting approx. 450,000 residents of the city

MSC Seminar (January 30, 2012)
Consequence of Salinity

- The salinity is more a matter of **taste** (increased content of chloride) than **health** while it is less than 2000 ppm

<table>
<thead>
<tr>
<th>Salinity level</th>
<th>Taste</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 - 300 ppm</td>
<td>Very slight saline taste</td>
</tr>
<tr>
<td>&gt;300 - 600 ppm</td>
<td>Noticeable discomfort in taste</td>
</tr>
<tr>
<td>&gt;600 - 800 ppm</td>
<td>Increasing discomfort in taste</td>
</tr>
<tr>
<td>&gt;800 - 1000 ppm</td>
<td>Very strong discomfort in taste</td>
</tr>
<tr>
<td>&gt;1000 - 2000 ppm</td>
<td>Extremely strong discomfort in taste</td>
</tr>
</tbody>
</table>

W.H.O. Guidelines

- Average dietary intake of chloride for human
  - Ranging from 6 g/day to 12 g/day
- The consequence is that daily intake of salt from water is usually **less than** 5% to 10% of total intake from foods
- **No evidence** on health effect of prolonged intake of large amounts of chloride in diet
  - Except in the special case of impaired sodium chloride metabolism, e.g. in congestive heart failure
Why not Supply Restriction?

- No health risk
- Restriction may cause sanitary problem with reduction on people hygiene
- May increase water duration in pipes, flush some particles, create vacuuming and potentially lead to bacteriological contamination
- May affect the overall economy of the city
- May damage International Image of the city

Tackling the Problem

- On the technical/engineering level
  - Improved monitoring and pumping system
  - Preparation before the crisis: top-up of reservoirs
  - Leak detection to reduce water loss
- Technical communication and coordination with related partners when the crisis started
- A software to optimize the logistical operations, so as to forecast and control the salinity of potable water below a desirable level
Objective

To maximize the number of days in which the potable salinity level is below the desirable level
## Existing Approach

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pump X Capacity</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Pump Y Capacity</td>
<td>1500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Pump Z Capacity</td>
<td>1500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Reservoir A Maximum volume</td>
<td>1300000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Reservoir A Minimum volume</td>
<td>3000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Reservoir B Maximum volume</td>
<td>1200000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Reservoir B Minimum volume</td>
<td>1000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Reservoir C Maximum volume</td>
<td>2400000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Reservoir C Minimum volume</td>
<td>1100000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Reservoir D Maximum volume</td>
<td>2000000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>River Supply</th>
<th>No. of Pump</th>
<th>Reservoir A</th>
<th>Reservoir B</th>
<th>Reservoir C</th>
<th>Consumed</th>
<th>Pottable</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/1/2005</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/2/2005</td>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/3/2005</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/4/2005</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/5/2005</td>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/6/2005</td>
<td>1200</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/7/2005</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
<tr>
<td>4/8/2005</td>
<td>1000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150,000</td>
</tr>
</tbody>
</table>

**Constants**
- Pump X Capacity: 3000
- Pump Y Capacity: 1500
- Pump Z Capacity: 1500
- Reservoir A Maximum volume: 1300000
- Reservoir A Minimum volume: 3000000
- Reservoir B Maximum volume: 1200000
- Reservoir B Minimum volume: 1000000
- Reservoir C Maximum volume: 2400000
- Reservoir C Minimum volume: 1100000
- Reservoir D Maximum volume: 2000000

**Given Data**
- River Supply: 1000, 1200, 1000, 1200
- No. of Pump: 0, 0, 0, 0
- Reservoir A: 0, 0, 0, 0
- Reservoir B: 0, 0, 0, 0
- Reservoir C: 0, 0, 0, 0
- Consumed: 150,000, 150,000, 150,000, 150,000
- Pottable: 30, 30, 30, 30

**Plot Diagram**
- Reservoirs A, B, C, D
- Pumps X, Y, Z
- Consumption

---

MSC Seminar (January 30, 2012)
## Existing Approach

### Manual Adjustment

<table>
<thead>
<tr>
<th>Pump</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir A</td>
<td>120,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reservoir B</td>
<td></td>
<td>120,000</td>
<td></td>
</tr>
<tr>
<td>Reservoir C</td>
<td></td>
<td></td>
<td>120,000</td>
</tr>
<tr>
<td>Reservoir D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Check Objective

| Date   | River Salinity | No. of Pump | Reservoir A | Reservoir A | Reservoir B | Reservoir B | Reservoir C | Reservoir C | Reservoir C | Reservoir D | Reservoir D | Consumption | Potable |
|--------|---------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------|--------|
| 4/6/0005 | 1000          | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 1000        | 1000      |
| 4/7/0005 | 1000          | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 1000        | 1000      |
| 4/8/0005 | 1000          | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 1000        | 1000      |
| 4/9/0005 | 1000          | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 1000        | 1000      |
Weaknesses

- Manual trial-and-error is **tedious and time consuming**

- Problem is **too large and too complex** for such manual optimization process, and the model is **overly simplified**

- Require an experienced operator
  - Knowledge to use a spreadsheet
  - Domain knowledge of the logistical operations

MSC Seminar (January 30, 2012)
Project Goals

- Automate the optimization process and reduce errors
- More realistic model
- Even novice users can operate
- Generate better quality solutions in a shorter time

Outline

- Domain Description
- Constraint Programming (CP)
- Problem Modelling
- Improvements
- Concluding Remarks
Real-life Applications of CP

- Options trading
- Gate allocation to aircrafts in airports
- Selection and scheduling of observations performed by satellites
- DNA sequencing, construction of 3D models of proteins
- Locating faults in the circuits, computing circuit layouts, testing and verification of design
- Many other scheduling, planning and optimization problems

MSC Seminar (January 30, 2012)

Constraint Satisfaction Problems

A CSP is a triple $< Z, D, C >$

- $Z$ is a finite set of variables $\{x_1, x_2, \ldots, x_n\}$
- $D$ is a finite set $\{D_1, D_2, \ldots, D_n\}$, where $D_i$ is a finite set of possible values for variable $x_i$
- $C$ is a set of constraints, each on a subset of $Z$ limiting the possible combinations of values that the variables can take

Goal: to find a consistent variable assignment

MSC Seminar (January 30, 2012)
Constrained Optimization

<table>
<thead>
<tr>
<th>Goods</th>
<th>Wine</th>
<th>Perfume</th>
<th>Cigarettes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>4 units</td>
<td>3 units</td>
<td>2 units</td>
</tr>
<tr>
<td>$$$</td>
<td>$15</td>
<td>$10</td>
<td>$7</td>
</tr>
</tbody>
</table>

- Constraint:
  
  \[ 4W + 3P + 2C \leq 9 \]
  
  \[ 15W + 10P + 7C \geq 30 \]

**Objective:** \[ 15W + 10P + 7C \]
Basic Branch & Bound

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 0

\[ 15*0 + 10*0 + 7*\max(P) = 0 + 0 + 28 = 28 \]

Basic Branch & Bound

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 0

\[ 15*0 + 10*0 + 7*\max(C) = 0 + 0 + 28 = 28 \]
Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 0

The subtree of $P=0$ (in green) is pruned, since $15*0 + 10*0 + 7*\max(C) < 30$.

Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 0

$15*0 + 10*1 + 7*\max(C) = 0 + 10 + 28 = 38$
Basic Branch & Bound

Obj: $15W + 10P + 7C$
Current Best: 0

W=0

P=1

The subtree of $C=4$ (in green) is pruned, since $4*0 + 3*1 + 2*4 > 9$.
Now $\max(C) = 3$

Obj: $15W + 10P + 7C$
Current Best: 0

W=0

P=1

$15*0 + 10*1 + 7*\max(C) = 0 + 10 + 21 = 31$
Basic Branch & Bound

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 0

\[ W = 0 \]
\[ P = 1 \]
\[ C = 1 \]

\[ 15*0 + 10*1 + 7*1 = 0 + 10 + 7 = 17 < 30 \]

\[ 4W + 3P + 2C \leq 9 \]
\[ 15W + 10P + 7C \geq 30 \]
Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 0

$15*0 + 10*1 + 7*2 = 0 + 10 + 14 = 24 < 30$

$15*0 + 10*1 + 7*3 = 0 + 10 + 21 = 31$

Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 0

$15*0 + 10*1 + 7*2 = 0 + 10 + 14 = 24 < 30$

$15*0 + 10*1 + 7*3 = 0 + 10 + 21 = 31$
Basic Branch & Bound

Obj: $15W + 10P + 7C$
Current Best: 31

Since $obj >$ current best value, current value is updated to 31.

Basic Branch & Bound

Obj: $15W + 10P + 7C$
Current Best: 31

$15*0 + 10*1 + 7*3 = 0 + 10 + 21 = 31$

$4W + 3P + 2C \leq 9$
$15W + 10P + 7C \geq 30$

$15*0 + 10*2 + 7*\max(C) = 0 + 20 + 28 = 48$
Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 31

W=0

P=2

15*0 + 10*2 + 7*max(C) = 0 + 20 + 28 = 48

The subtree of $C \geq 2$ (in green) is pruned, since $4*0 + 3*2 + 2*2 > 9$. Now, $max(C) = 1$.

Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 31

W=0

P=2

15*0 + 10*2 + 7*max(C) = 0 + 20 + 7 = 27

Since maximum possible obj is smaller than the current best value, the subtree $C=0$ and $C=1$ are pruned.
Basic Branch & Bound

**Obj:** $15W + 10P + 7C$

Current Best: 31

$15\times 0 + 10\times 3 + 7\times \max(C) = 0 + 30 + 28 = 58$

W=0

P=3

The subtree of $C \geq 1$ (in green) is pruned, since $4\times 0 + 3\times 3 + 2\times 1 > 9$.

Now, $\max(C) = 0$. 

15+0+10+3+7*max(C) = 0 + 30 + 28 = 58
Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 31

Since maximum possible obj is smaller than the current best value, the subtree $C=0$ is pruned.

Obj: $15W + 10P + 7C$

Current Best: 31

$15*0 + 10*3 + 7*\max(C) = 0 + 30 + 0 = 30$

$W=0$

$P=3$

$4W + 3P + 2C \leq 9$

$15W + 10P + 7C \geq 30$

$15*0 + 10*3 + 7*\max(C) = 0 + 30 + 0 = 30$

$W=1$

$P=3$

$15*1 + 10*\max(P) + 7*\max(C) = 15 + 30 + 28 = 73$
Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 31

$15 \times 1 + 10 \times 0 + 7 \times \max(C) = 15 + 0 + 28 = 33$

W=1

P=0

The subtree $P \geq 2$ is pruned, since $4 \times 1 + 3 \times 2 > 9$. 

Basic Branch & Bound

Obj: $15W + 10P + 7C$

Current Best: 31

$15 \times 1 + 10 \times 0 + 7 \times \max(C) = 15 + 0 + 28 = 33$
Basic Branch & Bound

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 31

\[ 15*1 + 10*0 + 7*\max(C) = 15 + 0 + 28 = 33 \]

The subtree \( C \geq 3 \) is pruned, since \( 4*1 + 3*0 + 2*3 > 9 \).

Now, \( \max(C) = 2 \).

---

Basic Branch & Bound

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 31

\[ 15*1 + 10*0 + 7*\max(C) = 15 + 0 + 14 = 29 \]

Since maximum possible \( \text{obj} \) < current best value, the subtree is prune.
Basic Branch & Bound

**Obj:** $15W + 10P + 7C$

Current Best: 31

The subtree $C \geq 2$ is pruned, since $4 \cdot 1 + 3 \cdot 1 + 2 \cdot 2 > 9$.

Now, $\max(C) = 1$. 

$15 \cdot 1 + 10 \cdot 1 + 7 \cdot \max(C) = 15 + 10 + 28 = 53$
Basic Branch & Bound

**Obj:** $15W + 10P + 7C$

Current Best: 31

$15*1 + 10*1 + 7*0 = 15 + 10 + 0 = 25$

$15*1 + 10*1 + 7*1 = 15 + 10 + 7 = 32$

$4W + 3P + 2C \leq 9$

$15W + 10P + 7C \geq 30$
Basic Branch & Bound

**Obj**: $15W + 10P + 7C$

Current Best: 32

- $15 \times 1 + 10 \times 1 + 7 \times 1 = 15 + 10 + 7 = 32$

Since $obj >$ current best value, current value is updated to 32.

---

**Obj**: $15W + 10P + 7C$

Current Best: 32

- $15 \times 2 + 10 \times \max(P) + 7 \times \max(C) = 30 + 30 + 28 = 88$

---

Basic Branch & Bound

**Obj**: $15W + 10P + 7C$

Current Best: 32

- $15W + 10P + 7C \geq 30$

- $4W + 3P + 2C \leq 9$
Basic Branch & Bound

Obj: $15W + 10P + 7C$
Current Best: 32

$15*2 + 10*\max(P) + 7*\max(C) = 30 + 30 + 28 = 88$

The subtree $P \geq 1$ is pruned, since $4*2 + 3*1 > 9$. Now $\max(P)=0$.

Obj: $15W + 10P + 7C$
Current Best: 32

$15*2 + 10*0 + 7*\max(C) = 30 + 0 + 28 = 58$
Basic Branch & Bound

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 32

\[ 15 \times 2 + 10 \times 0 + 7 \times \max(C) = 30 + 0 + 28 = 58 \]

W=2

P=0

The subtree \( C \geq 1 \) is pruned, since \( 4 \times 2 + 3 \times 0 + 2 \times 1 > 9 \). Now \( \max(C) = 0 \).

\[ \text{Obj: } 15W + 10P + 7C \]

Current Best: 32

\[ 4W + 3P + 2C \leq 9 \]

\[ 15W + 10P + 7C \geq 30 \]

W=2

P=0

Since \( \text{obj} < \) current best value, subtree \( C=0 \) is also pruned.
Search Efficiency

- **Variable Ordering**
  - Affects the shape and size of the search tree
  - Is first-fail principle all??

- **Value Ordering**
  - Affects the ordering of the branches
  - Move (good) solution branches as far to the left as possible (depth-first search from left to right)

- Constraint propagation: the more the better!
Outline

• Domain Description
• Constraint Programming (CP)
• Problem Modelling
• Improvements
• Concluding Remarks

Raw Water System Model

MSC Seminar (January 30, 2012)
Assumptions

• Salinity concentration in each reservoir is **homogeneous** and **instantaneous mixing** occurs when water is poured in each reservoir.

• Overflowing of reservoirs (to dilute) is **NOT** allowed.

• The time period is at most **180 days**.
Inputs (Parameters)

- Desirable potable salinity level \((C_{desire})\) and absolutely unacceptable potable salinity level \((C_{Max})\)
- Limit on the daily increase of potable salinity level \((DailyIncLimit)\)
- Initial volume and salinity levels of various reservoirs \((VA[0], CA[0], \ldots)\)
- Capacity of various reservoirs and pumps \((CapacityPX, \ldots, MinVolA, MaxVolA, \ldots)\)
- Limit on the water volume flow out of various reservoirs (i.e. size of pipes, gravity) \((MaxFlowA, \ldots)\)

Inputs (Problem Data)

- Input data for each day of the 90-day period
  - Salinity levels of the river water from pump Station X \((CX[i])\)
  - Daily water consumption pattern of the city \((V_{consume}[i])\)
  - Controlled threshold level for the various reservoirs \((ThresholdA[i], \ldots)\)
Decision Variables

The number of pumping hours at Station X on the $i^{th}$ day ($PX[i]$)

The number of pumping hours at Station Y on the $i^{th}$ day ($PY[i]$)
Decision Variables

The number of pumping hours at Station Z on the \( i \)th day (\( PZ[i] \))

Decision Variables

The outflow volume of Reservoir A on the \( i \)th day (\( VoutA[i] \))
### Decision Variables

- **Pump X**
- **Pump Y**
- **Pump Z**
- **Reservoir A**
- **Reservoir B**
- **Reservoir C**
- **Reservoir D**

**Consumption**

The outflow volume of Reservoir B on the $i^{th}$ day ($V_{outB[i]}$)

The outflow volume of Reservoir C on the $i^{th}$ day ($V_{outC[i]}$)
Decision Variables

Domain Discretization

- Domain of the Decision variables (pumping hours and water transfer) are continuous in nature
- Discretize and quantize the domain to reduce the search space
  - PumpQuanta
    - Pump usage quanta/unit (e.g. 3 hrs or 6 hrs)
  - TransferQuanta
    - Water transfer quanta/unit (e.g. 5,000m³ or 10,000m³)
### Domains of Decision Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>PX[i]</td>
<td>(0 \ldots \frac{\text{NumOfPumpX} \times 24}{\text{PumpQuanta}})</td>
</tr>
<tr>
<td>PY[i]</td>
<td>(0 \ldots \frac{\text{NumOfPumpY} \times 24}{\text{PumpQuanta}})</td>
</tr>
<tr>
<td>PZ[i]</td>
<td>(0 \ldots \frac{\text{NumOfPumpZ} \times 24}{\text{PumpQuanta}})</td>
</tr>
<tr>
<td>VoutA[i]</td>
<td>(0 \ldots \frac{\text{MaxFlowA}}{\text{TransferQuanta}})</td>
</tr>
<tr>
<td>VoutB[i]</td>
<td>(0 \ldots \frac{\text{MaxFlowB}}{\text{TransferQuanta}})</td>
</tr>
<tr>
<td>VoutC[i]</td>
<td>(0 \ldots \frac{\text{MaxFlowC}}{\text{TransferQuanta}})</td>
</tr>
<tr>
<td>VoutD[i]</td>
<td>(0 \ldots \frac{\text{MaxFlowD}}{\text{TransferQuanta}})</td>
</tr>
</tbody>
</table>

### Auxiliary Variables

- Reservoir A
- Reservoir B
- Reservoir C
- Reservoir D
- Pump X
- Pump Y
- Pump Z

The volume of water pump from station X on the \(i\)th day (VPX[i])

MSC Seminar (January 30, 2012)
The water volume of Reservoir A on the $i^{th}$ day ($V_A[i]$)

The salinity level of Reservoir A on the $i^{th}$ day ($C_A[i]$)
Auxiliary Variables

The water intake of Reservoir A on the $i$th day ($\text{VinA}[i]$)

The water volume of Reservoir B on the $i$th day ($\text{VB}[i]$)
The salinity level of Reservoir B on the $i^{th}$ day ($CB[i]$)

The water intake of Reservoir B on the $i^{th}$ day ($VinB[i]$)

MSC Seminar (January 30, 2012)
Auxiliary Variables

The water volume of Reservoir C on the $i^{th}$ day ($VC[i]$)

The salinity level of Reservoir C on the $i^{th}$ day ($CC[i]$)
The water intake of Reservoir C on the $i^{th}$ day ($V_{inC[i]}$)

Volume of water remained after water pumped to Reservoirs A and B on the $i^{th}$ day ($Temp[i]$)
The water volume of Reservoir D on the ith day ($VD[i]$)

The salinity level of Reservoir D on the ith day ($CD[i]$)
The water intake of Reservoir D on the ith day \((V_{inD[i]})\)

Volume of water remained on \(V_{outC[i]}\) after transfer to Reservoir D \((V_{restC[i]}))\)
Constraints

- Law of conservation of water and salts
- Physical Limitations
- Consumer Satisfaction

First Type of Constraints

- Law of conservation of water of a reservoir (linear relation)

\[
\begin{align*}
\text{Reservoir Volume day (i)} & = \text{Reservoir Volume day (i-1)} - \text{Volume flow-out day (i)} + \text{Volume flow-in day (i)} \\
\end{align*}
\]

- Law of conservation of salts of a reservoir (nonlinear relation)

\[
\begin{align*}
\text{Reservoir Volume day (i)} \times \text{Reservoir Salinity day (i)} & = \text{Reservoir Volume day (i-1)} \times \text{Reservoir Salinity day (i-1)} - \\
\text{Volume flow-out day (i)} \times \text{Reservoir Salinity day (i-1)} & + \text{Volume flow-in day (i)} \times \text{Salinity flow-in (2012)}
\end{align*}
\]
Conservation of Water for Pump X:
\[ VPX[i] = PX[i] \times \text{PumpQuanta} \times \text{CapacityPX} \]

Conservation of Water for Pump Y:
\[ VinA[i] = PY[i] \times \text{PumpQuanta} \times \text{CapacityPY} \]
Conservation of Water for Pump Z:
VinB[i] = PZ[i] * PumpQuanta * CapacityPZ

Conservation of water for Reservoir A:
Conservation of water for Reservoir B:
\[ V_{B[i]} = V_{B[i-1]} + V_{inB[i]} - (V_{outB[i]} \times \text{TransferQuanta}) \]

Conservation of water:
\[ \text{Temp}[i] = \text{VPX}[i] - V_{inA[i]} - V_{inB[i]} \]
Conservation of water:
\[ \text{VinC}[i] = \text{Temp}[i] + (\text{VoutA}[i] \times \text{TransferQuanta}) + (\text{VoutB}[i] \times \text{TransferQuanta}) \]

Conservation of water for Reservoir C:
\[ \text{VC}[i] = \text{VC}[i-1] + \text{VinC}[i] - (\text{VoutC}[i] \times \text{TransferQuanta}) \]
Conservation of water:

VinD[i] = (VoutC[i] * TransferQuanta) – VrestC[i]

Conservation of water for Reservoir D:

VD[i] = VD[i-1] + VinD[i] – (VoutD[i] * TransferQuanta)
Conservation of water for consumption:
\[ V_{\text{consume}[i]} = V_{\text{restC}[i]} + (V_{\text{outD}[i]}) \times \text{TransferQuanta} \]

Conservation of salt for Reservoir A:
\[ (C_{A[i]} \times V_{A[i]}) = (C_{A[i-1]} \times V_{A[i-1]}) + (C_{X[i]} \times V_{inA[i]}) - (C_{A[i-1]} \times V_{outA[i]} \times \text{TransferQuanta}) \]
Conservation of salt for Reservoir B:
\[(CB[i] \times VB[i]) = (CB[i-1] \times VB[i-1]) + (CX[i] \times VinB[i]) - (CB[i-1] \times VoutB[i] \times TransferQuanta)\]

Conservation of salt for Reservoir C:
\[(CC[i] \times VC[i]) = (CC[i-1] \times VC[i-1]) + (CX[i] \times Temp[i]) + (CA[i-1] \times VoutA[i] \times TransferQuanta) + (CB[i-1] \times VoutB[i] \times TransferQuanta) - (CC[i-1] \times VoutC[i] \times TransferQuanta)\]
Conservation of salt for Reservoir D:
\[(\text{CD}[i] \times \text{VD}[i]) = (\text{CD}[i-1] \times \text{VD}[i-1]) + (\text{CC}[i-1] \times \text{VinD}[i]) \]
\[\quad - (\text{CD}[i-1] \times \text{VoutD}[i] \times \text{TransferQuanta})\]
Second Type of Constraints

- Physical limitations on the capacity of pumps, reservoirs and pipes (linear relation)
- Water flows from Reservoir C to D by gravity (nonlinear relation)

<table>
<thead>
<tr>
<th>Reservoir C Capacity (m$^3$)</th>
<th>Flow Limit (m$^3$/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,345,650-2,454,590</td>
<td>211,395</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>1,200,000-1,256,250</td>
<td>160,445</td>
</tr>
</tbody>
</table>

Constraints

Physical Limitation:
Constraints

MSC Seminar (January 30, 2012)

Physical Limitation:

MSC Seminar (January 30, 2012)

Physical Limitation:
MinVolC ≤ VC[i] ≤ MaxVolC, VC[i] ≥ ThresholdC[i]
Constraints

Physical Limitation:

Physical Limitation:
VoutA[i] ≤ MaxFlowA
Constraints

 Physical Limitation: \( V_{outB[i]} \leq MaxFlowB \)

Water transfer from Reservoir C to D by gravity

MSC Seminar (January 30, 2012)
Constraints

Physical Limitation:

<table>
<thead>
<tr>
<th>Reservoir C Capacity (m³)</th>
<th>Flow Limit (m³/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,345,650-2,454,590</td>
<td>211,395</td>
</tr>
<tr>
<td>2,236,710-2,345,650</td>
<td>202,503</td>
</tr>
<tr>
<td>1,256,250-1,365,190</td>
<td>164,790</td>
</tr>
<tr>
<td>1,200,000-1,256,250</td>
<td>160,445</td>
</tr>
</tbody>
</table>

Constraints

Physical Limitation:

\[ \text{VoutD}[i] \leq \text{MaxFlowD} \]
Third Type of Constraints

- Consumer Satisfactions \((\text{linear relation})\)

\[
\begin{align*}
\text{Potable Salinity day (i)} & \leq \text{Desirable Salinity Level} \\
\text{Potable Salinity day (i)} & \leq \text{Potable Salinity day (i-1)} + \text{Salinity Increase Limit}
\end{align*}
\]

Constraints

\[
\begin{align*}
\text{Reservoir A} & \rightarrow \text{Pump X} \rightarrow \text{Reservoir B} \\
\text{Reservoir B} & \rightarrow \text{Pump Y} \rightarrow \text{Reservoir C} \\
\text{Reservoir C} & \rightarrow \text{Pump Z} \rightarrow \text{Reservoir D} \\
\text{Reservoir D} & \rightarrow \text{Consumption}
\end{align*}
\]

Consumer Satisfaction:
\(\text{Cpotable}[i] \leq \text{CMax}\)
Constraints

Consumer Satisfaction: $C_{\text{potable}[i]} \leq C_{\text{potable}[i-1]} + \text{DailyIncLimit}$

Objective

To maximize the number of days in which the potable salinity level is below the desirable level
Objective

\[
\text{Maximize } \sum (C_{\text{potable}}[i] \leq C_{\text{desire}})
\]

Implementation Platform

- State-of-the-art constraint programming system
  - ILOG Solver 6.0 (C++ library)
  - GNU Compiler Collection (GCC 3.2)
  - PC running GNU/Linux (Linux kernel 2.4)
Difficulties

• The problem size is large
  – Salinity periods of 180 days
  – About 4,500 variables and 9,000 constraints
  – About \((3,612,000)^{180}\) possible search states

• The problem involves both linear and non-linear constraints

MSC Seminar (January 30, 2012)

Difficulties

• The different components of the raw water system interact with each other temporally and geographically (topologically)

• Although efficient commercial optimization tools are available, out-of-the-box execution strategies fail to handle even small test cases

MSC Seminar (January 30, 2012)
Scenario A

Default Search Heuristic
Result: Can handle only a 30-day period!!

All Reservoirs Drained!!!!!!!!!!!
Outline

• Domain Description
• Constraint Programming (CP)
• Problem Modelling
• Improvements
• Concluding Remarks

MSC Seminar (January 30, 2012)

Improvements - 1

• Specialized variable ordering heuristics
  – Affects the shape and size of the search tree
  – Is first-fail principle all??

• Mimic how human operators would go about labeling the variables manually

• Time-consuming but rewarding!!

• Best we have so far after some experiments

MSC Seminar (January 30, 2012)
Improvements - 2

- Specialized value ordering heuristics
  - Affects the ordering of the branches
  - Move (good) solution branches as far to the left as possible (depth-first search from left to right)

- Control how much to pump from the river

- Control how much to transfer between reservoirs (which reservoir’s water to use first)

MSC Seminar (January 30, 2012)

Improvements - 3

- Changing constraint representations by adding auxiliary variables and rewriting the constraints

- Induce extra constraint propagation and thus also pruning: the more the better!!

MSC Seminar (January 30, 2012)
Scenario A Revisited

Result: 90 days out of 90 days Below 250
Runtime ~ 7 sec

Scenario A

Result: 180 days out of 180 days Below 250
Runtime ~ 30 sec
Scenario B

Result: 117 days out of 180 days Below 250

Runtime ~ 1 min

Salinity Profiles

Scenario A
Scenario B
Scenario C
Consumption Profiles

Results for 90 Days (Scenario A)

<table>
<thead>
<tr>
<th>salinity</th>
<th>Normal</th>
<th>High Consumption</th>
<th>Old Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>desire</td>
<td>max</td>
<td>days  sec. fails</td>
<td>days  sec. fails</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>74 5 0</td>
<td>51 425 1</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>90 7 0</td>
<td>87 366 0</td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td>90 8 0</td>
<td>87 367 0</td>
</tr>
<tr>
<td>250</td>
<td>500</td>
<td>90 10 0</td>
<td>87 369 0</td>
</tr>
<tr>
<td>250</td>
<td>600</td>
<td>90 12 0</td>
<td>87 370 0</td>
</tr>
<tr>
<td>250</td>
<td>1000</td>
<td>90 19 0</td>
<td>87 377 0</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>90 12 0</td>
<td>90 11 1</td>
</tr>
<tr>
<td>300</td>
<td>1000</td>
<td>90 19 0</td>
<td>90 17 1</td>
</tr>
</tbody>
</table>
### Results for 90 Days (Scenario B)

<table>
<thead>
<tr>
<th>salinity</th>
<th>Normal</th>
<th>High Consumption</th>
<th>Old Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>desire</td>
<td>max</td>
<td>days sec. fails</td>
<td>days sec. fails</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>90    7 0</td>
<td>90    7 1</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>90    8 0</td>
<td>90    7 0</td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td>90    8 0</td>
<td>90    8 0</td>
</tr>
<tr>
<td>250</td>
<td>500</td>
<td>90    9 0</td>
<td>90    9 0</td>
</tr>
<tr>
<td>250</td>
<td>600</td>
<td>90    10 0</td>
<td>90   10 0</td>
</tr>
<tr>
<td>250</td>
<td>1000</td>
<td>90   16 0</td>
<td>90   16 0</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>90    10 1</td>
<td>90   10 1</td>
</tr>
<tr>
<td>300</td>
<td>1000</td>
<td>90   16 1</td>
<td>90   16 1</td>
</tr>
</tbody>
</table>

### Results for 90 Days (Scenario C)

<table>
<thead>
<tr>
<th>salinity</th>
<th>Normal</th>
<th>High Consumption</th>
<th>Old Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>desire</td>
<td>max</td>
<td>days sec. fails</td>
<td>days sec. fails</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>--   -- --</td>
<td>--   -- --</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>--   -- --</td>
<td>--   -- --</td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td>--   -- --</td>
<td>21   2403 198</td>
</tr>
<tr>
<td>250</td>
<td>500</td>
<td>--   -- --</td>
<td>28   1803 0</td>
</tr>
<tr>
<td>250</td>
<td>600</td>
<td>--   -- --</td>
<td>28   1804 0</td>
</tr>
<tr>
<td>250</td>
<td>1000</td>
<td>--   -- --</td>
<td>28   1805 0</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>--   -- --</td>
<td>45   1804 1</td>
</tr>
<tr>
<td>300</td>
<td>1000</td>
<td>--   -- --</td>
<td>45   1805 1</td>
</tr>
</tbody>
</table>
### Results for 180 Days (Scenario A)

<table>
<thead>
<tr>
<th>Salinity</th>
<th>Normal</th>
<th>High Consumption</th>
<th>Old Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>desire</td>
<td>max</td>
<td>days sec. fails</td>
<td>days sec. fails</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>157 23 3</td>
<td>126 560 3</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>180 25 3</td>
<td>168 503 2</td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td>180 27 3</td>
<td>168 385 2</td>
</tr>
<tr>
<td>250</td>
<td>500</td>
<td>180 32 3</td>
<td>168 449 2</td>
</tr>
<tr>
<td>250</td>
<td>600</td>
<td>180 38 3</td>
<td>168 453 2</td>
</tr>
<tr>
<td>250</td>
<td>1000</td>
<td>180 61 3</td>
<td>168 471 2</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>180 40 3</td>
<td>180 35 4</td>
</tr>
<tr>
<td>300</td>
<td>1000</td>
<td>180 68 3</td>
<td>180 53 4</td>
</tr>
</tbody>
</table>

### Results for 180 Days (Scenario B)

<table>
<thead>
<tr>
<th>Salinity</th>
<th>Normal</th>
<th>High Consumption</th>
<th>Old Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>desire</td>
<td>max</td>
<td>days sec. fails</td>
<td>days sec. fails</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>-- -- --</td>
<td>-- -- --</td>
</tr>
<tr>
<td>250</td>
<td>350</td>
<td>-- -- --</td>
<td>-- -- --</td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td>-- -- --</td>
<td>-- -- --</td>
</tr>
<tr>
<td>250</td>
<td>500</td>
<td>106 2427 1</td>
<td>-- -- --</td>
</tr>
<tr>
<td>250</td>
<td>600</td>
<td>110 1831 1</td>
<td>108 1227 3</td>
</tr>
<tr>
<td>250</td>
<td>1000</td>
<td>118 773 2</td>
<td>131 835 24119</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>124 1832 1</td>
<td>129 1228 7</td>
</tr>
<tr>
<td>300</td>
<td>1000</td>
<td>134 1433 1</td>
<td>136 687 4340</td>
</tr>
</tbody>
</table>
Advantages

• Formulate a model relatively close to the original problem, making the model easy to verify and maintain
• Design domain specific search heuristic to reduce the time of searching for solutions
• Find better quality solutions in a much shorter time than human operators
• Can be used by novice operators

Outline

• Domain Description
• Constraint Programming (CP)
• Problem Modelling
• Improvements
• Concluding Remarks
Genetic Algorithms

- UNU-IIST tried EVOLVER, which is a GA-based optimization engine for MS Excel
- Less efficient and lower quality solution
- Semi-automatic: requires expert human guidance during search
- Unstable and unpredictable with regard to convergence

MSC Seminar (January 30, 2012)

Mathematical Programming

- Collaboration with our OR experts
- Advantage: the domain of the problem is continuous in nature (i.e. real numbers)
- Major difficulties: non-linear constraints
  - Law of conservation of salts
  - Table constraints

MSC Seminar (January 30, 2012)
Concluding Remarks

- Introduced to the problem in October, 2004
- First prototype early December, 2004
- Version 3.x now
- 1 PhD student and 2 final year undergraduates (+ Me)
- User acceptance passed

MSC Seminar (January 30, 2012)

Concluding Remarks

- We apply CP to solve the optimization of logistical operations during the salinity periods (to the best of our knowledge, the first application of CP to water resource management)
- The system is expected to benefit some 450,000 residents during the upcoming salinity periods

Still, CP cannot combat with nature!

MSC Seminar (January 30, 2012)
The Web Interface

Welcome to the Salinity Optimization Engine!

The Salinity Optimization Engine helps you in finding optimized operations for the Drinking Water system. It tries to keep the salinity of the drinking water below a desirable level over a specified period of time.

There are two methods to start the optimizer:

Method 1: To specify the values for the input parameters in each field under the sections: Salinity, Running Period, Initial Data of Reservoir, Reservoir Capacity, Pump and Others.

Method 2: To click on the checkbox and upload the two input files under the Files section.

Collaboration with UNU-IIST

- Core optimization engine developed by CUHK
- Web interface developed by UNU-IIST

UNU-IIST
International Institute for Software Technology

MSC Seminar (January 30, 2012)