## Final Exam

Due on Dec 31. Send your solution to me. If you use email, please cc it to the TA.

1. **[Communication]** Consider the *k*-player Number-in-Hand model (see Lecture 4). Define functions

and

What are the deterministic communication complexities of these two functions?

1. **[Formula]** Define the sensitivity of a Boolean function on input x as where is the string obtained from by flipping the *i*-th bit. Further define measures

 and .

Prove that Khrapchenko’s bound (Theorem 2.2 in Lecture 5) cannot prove lower bound larger than .

1. **[Fourier]** Recall that for a Boolean function , the influence of variable is defined as , where is the as defined in the previous question. Show that . When does the equality hold?
2. **[Decision Tree]** Suppose that we have two functions, , with deterministic decision tree complexity and , respectively. Now we want to compute a new function defined by (the AND of the two functions). What’s the deterministic decision tree complexity of ?
3. **[Circuit]** We proved a lower bound of for the size of depth-d circuit for computing the Parity function (Theorem 2.1 and the Corollary after it in Lecture 10). Do you think we can use the same method to prove a similar lower bound for the Majority function? What’s the best lower bound you can prove for Majority?