Homework 2

Due at 5pm, Apr 2, 2015.

Problem 1.

1. Recall that a subsequence of string A is a sequence which can be obtained by deleting some elements from A without changing the order. A common subsequence between two strings A and B is a subsequence for both A and B. Consider the following problem: Given two strings $A = a_1a_2 \dots a_n$ and $B = b_1b_2 \dots b_m$, find the maximum length of a common subsequence between A and B. Please give an algorithm with time complexity $O(nm)$.
   (*Hint: Dynamic programming, similar to Edit Distance.*)
2. (*exercise: no need to turn in*) A sequence $a_1a_2 \dots a_n$ is symmetric if $a_1 = a_n$, $a_2 = a_{n-1}$, ..., $a_{\lfloor (n+1)/2 \rfloor} = a_{\lceil (n+1)/2 \rceil}$. Design an algorithm to solve the following task: Given a string $A = a_1a_2 \dots a_n$, find a longest symmetric subsequence.

Problem 2. Formulate the maximum network flow problem as a linear program.

Problem 3.

1. Recall the following rule to write down the dual for LP above.

$$\begin{array}{lll} \max & c^T x & \\ s.t. & Ax = b & \\ & Dx \le e & \\ & x \ge 0 & \end{array} \quad \Longrightarrow \quad \begin{array}{lll} \min & b^T y + e^T z & \\ s.t. & A^T y + D^T z \ge c & \\ & z \ge 0 & \end{array}$$

   Now write down the dual of the LP that you wrote for the previous question.
2. (*exercise: no need to turn in*) Can you interpret the obtained dual LP as a *s*-*t* min-cut problem?

Problem 4. Design a divide-and-conquer algorithm to solve the following problem in time $O(n \log n)$: Given an integer array $A[1 \dots n]$, compute

$$|\{|(i,j) : i < j \text{ and } A[i] > A[j]\}|.$$

(*Hint: Relate this task to merge sort.*)