1. For any integer $k \geq 0$, define $L_k = \{ww \mid w \in \{0, 1\}^k\}$.
   (a) Write down all strings in $L_3$.
   (b) Prove that any DFA for $L_k$ has at least $2^k$ states.
       Hint: After reading the first half of the input, what should the DFA remember? Can you come up with a set of $2^k$ strings that are pairwise distinguishable by $L_k$?

2. Let $L$ be any language. We say that two strings $x$ and $y$ are indistinguishable by $L$ if for every string $z$, we have $xz \in L$ if and only if $yz \in L$.
   (a) For concreteness, consider $L_1 = \{x \in \{0, 1\}^* \mid$ the number of 1’s in $x$ is divisible by 3$\}$. Prove that 1 and 1111 are indistinguishable by $L_1$.
   (b) Continuing with (a), which strings are indistinguishable from the string 1 by $L_1$? The set of all such strings is the equivalence class of the string 1 and will be denoted by $[1]$.
   (c) Find a string $s$ not in $[1]$. What is the equivalence class of $s$? (We will denote this equivalence class by $[s]$)
   (d) Can you find another string $t$ not in $[1]$ or $[s]$? What is the equivalence class of $t$?
   (e) Can you find yet another string $u$ not in these equivalence classes?
   (f) Design a DFA for the language $L_1$. How are the states in your DFA related to the equivalence classes?