1. For any integer $k \geq 0$, define $L_k = \{ww \mid w \in \{0, 1\}^k\}$.

   (a) Write down all strings in $L_3$.

   (b) Prove that any DFA for $L_k$ has at least $2^k$ states.

   Hint: After reading the first half of the input, what should the DFA remember? Can you come up with a set of $2^k$ strings that are pairwise distinguishable by $L_k$?

2. For an integer $k \geq 1$, define $L_k$ to be the set of strings (over $\Sigma = \{0, 1\}$) that have a 1 at the $k$th-to-last position. For example, 100 and 01101 are in $L_3$, but 0 and 011 are not.

   (a) Prove that every DFA for $L_k$ has at least $2^k$ states.

   (b) Describe (e.g. with a diagram) an NFA for $L_k$ that has at most $k + 1$ states.

3. Let $L$ be the set of strings over $\{0, 1\}$ whose number of ones is a perfect square (e.g. 0, 1, 4, 9, 16, …). Prove that $L$ is irregular.