1. (a) Write down a regular expression for the following NFA. For this problem, you do not
have to go through the procedure described in class.
(b) Convert the following NFA into a DFA.

2. (a) Write down the definition of regular expressions over an alphabet Σ.
(b) Given a string w, define \( w^R \) as the string w in reverse order. That is, if \( w = w_1w_2...w_n \), then \( w^R = w_nw_{n-1}...w_1 \). For example, if \( w = \text{live} \), then \( w^R = \text{evil} \).

Given a language \( L \), define its reversal \( L^R \) as the set of strings in \( L \) in reverse. More
precisely, \( L^R = \{ w^R \mid w \in L \} \). For example, if \( L = \{ \text{live}, \text{raw}, \text{level} \} \), then
\( L^R = \{ \text{evil}, \text{war}, \text{level} \} \).

If \( L \) is a regular language, prove that \( L^R \) as also regular.

3. Let \( L \) be any language. We say that two strings \( x \) and \( y \) are indistinguishable by \( L \) if for
every string \( z \), we have \( xz \in L \) if and only if \( yz \in L \).

(a) For concreteness, consider \( L_1 = \{ x \in \{0, 1\}^* \mid \text{the number of 1's in } x \text{ is divisible by 3} \} \).
Prove that 1 and 1111 are indistinguishable by \( L_1 \).
(b) Continuing with (a), which strings are indistinguishable from the string 1 by \( L_1 \)?
The set of all such strings is the equivalence class of the string 1 and will be denoted
by \([1]\).
(c) Find a string \( s \) not in \([1]\). What is the equivalence class of \( s \)? (We will denote this
equivalence class by \([s]\))
(d) Can you find another string \( t \) not in \([1]\) or \([s]\)? What is the equivalence class of \( t \)?
(e) Can you find yet another string \( u \) not in these equivalence classes?
(f) Design a DFA for the language \( L_1 \). How are states in your DFA related to the
equivalence classes?