Week 3 Tutorial Session

Tutorial exercises include more problems than what a typical student can solve in 15–20 minutes. Don’t be discouraged if you cannot solve all the problems within the time limit.

1. (a) Write down a regular expression for the following NFA. For this problem, you do not have to go through the procedure described in class.

(b) Convert the following NFA into a DFA.

```
A
  | 0, 1
  v   
  0  A
      |   0
    v   v
  1  B  1
     |   |   0, 1
     v   v
  0  C  D
```

2. Let $L$ be any language. We say that two strings $x$ and $y$ are indistinguishable by $L$ if for every string $z$, we have $xz \in L$ if and only if $yz \in L$.

   (a) For concreteness, consider $L_1 = \{x \in \{0, 1\}^* \mid \text{the number of 1’s in } x \text{ is divisible by 3}\}$. Prove that 1 and 1111 are indistinguishable by $L_1$.

   (b) Continuing with (a), which strings are indistinguishable from the string 1 by $L_1$? The set of all such strings is the equivalence class of the string 1 and will be denoted by $[1]$.

   (c) Find a string $s$ not in $[1]$. What is the equivalence class of $s$? (We will denote this equivalence class by $[s]$)

   (d) Can you find another string $t$ not in $[1]$ or $[s]$? What is the equivalence class of $t$?

   (e) Can you find yet another string $u$ not in these equivalence classes?

   (f) Design a DFA for the language $L_1$. How are states in your DFA related to the equivalence classes?

3. (a) Write down the definition of regular expressions over an alphabet $\Sigma$.

   (b) Write down the definition of regular languages over an alphabet $\Sigma$.

   (c) Given a string $w$, define $w^R$ as the string $w$ in reverse order. That is, if $w = w_1w_2\ldots w_n$, then $w^R = w_nw_{n-1}\ldots w_1$. For example, if $w = \text{live}$, then $w^R = \text{evil}$.

   Given a language $L$, define its reversal $L^R$ as the set of strings in $L$ in reverse. More precisely, $L^R = \{w^R \mid w \in L\}$. For example, if $L = \{\text{live, raw, level}\}$, then $L^R = \{\text{evil, war, level}\}$.

   If $L$ is a regular language, prove that $L^R$ as also regular.