Week 3 Tutorial Session

1. (a) Write down a regular expression for the following NFA. For this problem, you do not have to go through the procedure described in class.

(b) Convert the following NFA into a DFA.

```
A --0, 1-- B
|       |
0       0, 1
|       |
A --1-- C
|       |
1       1
|       |
C --0-- D
```

2. Let $L$ be any language. We say that two strings $x$ and $y$ are indistinguishable by $L$ if for every string $z$, we have $xz \in L$ if and only if $yz \in L$.

(a) For concreteness, consider $L_1 = \{ x \in \{0, 1\}^* \mid$ the number of 1's in $x$ is divisible by 3}.
Prove that 1 and 1111 are indistinguishable by $L_1$.

(b) Continuing with (a), which strings are indistinguishable from the string 1 by $L_1$?
The set of all such strings is the equivalence class of the string 1 and will be denoted by $[1]$.

(c) Find a string $s$ not in $[1]$. What is the equivalence class of $s$? (We will denote this equivalence class by $[s]$)

(d) Can you find another string $t$ not in $[1]$ or $[s]$? What is the equivalence class of $t$?

(e) Can you find yet another string $u$ not in these equivalence classes?

(f) Design a DFA for the language $L_1$. How are states in your DFA related to the equivalence classes?

3. (a) Write down the definition of regular expressions over an alphabet $\Sigma$.

(b) Given a string $w$, define $w^R$ as the string $w$ in reverse order. That is, if $w = w_1w_2\ldots w_n$, then $w^R = w_nw_{n-1}\ldots w_1$. For example, if $w = \text{live}$, then $w^R = \text{evil}$.
Given a language $L$, define its reversal $L^R$ as the set of strings in $L$ in reverse. More precisely, $L^R = \{ w^R \mid w \in L \}$. For example, if $L = \{ \text{live, raw, level} \}$, then $L^R = \{ \text{evil, war, level} \}$.
If $L$ is a regular language, prove that $L^R$ as also regular.