Variants of Turing Machines

CSCI 3130 Formal Languages and Automata Theory

Siu On CHAN
Fall 2020

Chinese University of Hong Kong
Transitions may depend on the contents of all cells under the heads

Different tape heads can move independent
Multitape Turing machine

Multiple tapes are convenient
Some tapes can serve as temporary storage
Multitape Turing machines are **equivalent** to single-tape Turing machines.

- Multiple tapes can **easily** be simulated by a single tape.
- A single tape requires simulation to mimic the behavior of multiple tapes.

How to argue equivalence
Simulating multitape Turing machine

\[ \Gamma = \{a, b, \square\} \]

\[ \Gamma = \{a, b, \square, \dot{a}, \dot{b}, \#\} \]
Simulating multitape Turing machine

We show how to simulate a multitape Turing machine on a single tape Turing machine.

To be specific, let's simulate a 3-tape TM:

Multitape TM $M$

Single tape TM $S'$
Simulating multitape Turing machine

**Single-tape TM: Initialization**

$S$: On input $w_1 \ldots w_n$:

Replace tape contents by $\# \dot{w}_1 w_2 \ldots w_n \# \# \# \# \#$

Remember that $M$ is in state $q_0$
**Simulating multitape Turing machine**

**Single-tape TM:** Simulating multitape TM moves

Suppose Multitape TM $M$ moves like this:

```
b a □ □ ...
ab a □ ...
̃a a b □ ...
```

```
\begin{align*}
\text{□/□L} & \quad \text{a/bR} \\
\text{a/aR} & \\
\end{align*}
```

```
b a □ □ ...
```

```
\text{a b b □ ...}
```

```
\text{a a b □ ...}
```

We simulate the move on single-tape TM $S$ like this

```
# b a □ # a b a # ̃a a b # □
```

```
# b ̃a # a b b □ # a ̃a b # □
```

```
# b ̃a # a b b □ # a ̃a b # □
```
Simulating multitape Turing machine

Given input $w_1 \ldots w_n$:
Replace tape contents by $\# w_1 w_2 \ldots w_n \# \cdot \# \cdot$
Remember (in state) that $M$ is in state $q_0$

$S$ simulates a step of $M$:
Make a pass over tape to find $\dot{x}, \dot{y}, \dot{z}$

If $M$ at state $q_a$ has transition

\[
\begin{align*}
q_a & \xrightarrow{x/x' A} q_b \\
q_a & \xrightarrow{y/y' B} q_b \\
q_a & \xrightarrow{z/z' C} q_b
\end{align*}
\]

update state/tape accordingly

If $M$ reaches accept (reject) state, $S$ accepts (rejects)
To simulate a model $M$ by another model $N$:

Say how the state and storage of $N$ is used to represent the state and storage of $M$.

Say what should be initially done to convert the input of $N$.

Say how each transition of $M$ can be implemented by a sequence of transitions of $N$. 