(1) (40 points) Give a DFA for the following languages, specified by a transition diagram. For each one of them, give a short and clear description of how the machine works. The alphabet is $\Sigma = \{0, 1\}$ unless otherwise specified:

(a) $L_1 = \{w \mid w$ has at least two $1$s and at most one $0\}$.  
(b) $L_2 = \{w \mid$ the sum of digits of $w$ is divisible by 4\}.  
(c) $L_3$ is the language described by $2^*1^*0^*$. The alphabet is $\Sigma = \{0, 1, 2\}$.  
(d) $L_4 = \{w \mid w$ contains the substring 11 an odd number of times\}. Note that 111 contains two occurrences of 11.

(2) (10 points) Convert the following NFA to a DFA using the method described in class. Specify the DFA by its transition diagram. The alphabet is $\Sigma = \{0, 1\}$.

(3) (25 points) If $w$ is a string, we say that a string $x$ is an initial part of $w$ if $w = xy$ for some string $y$. For example, $b$ and $bcd$ are both initial parts of $bcde$. Given a language $L$, define $L^I = \{x \mid x$ is an initial part of some $w \in L\}$. That is, $L^I$ contains the initial parts of strings in $L$.

Prove that if $L$ is a regular language, then so is $L^I$.

Hint: Regular language is defined recursively. If the desired result is true for simpler regular languages, can you show that it is also true for more complex regular languages?
(4) (25 points) Consider the following DFA:

(a) What strings stop at $q_0$? At $q_1$? At $q_2$? At $q_3$?
(b) State an induction hypothesis that will allow you to prove your answer in (a).
(c) What is the language of the DFA?