Week 3 Tutorial Session

1. (a) Write down a regular expression for the following NFA. For this problem, you do not have to go through the procedure described in class.

(b) Convert the following NFA into a DFA.

```
\begin{array}{c}
q_0 \\
ε
\end{array} →
\begin{array}{c}
q_1 \\
ε
\end{array} →
\begin{array}{c}
q_2 \\
a
\end{array} →
\begin{array}{c}
q_3 \\
ε
\end{array} →
\begin{array}{c}
q_4 \\
b
\end{array} →
\begin{array}{c}
q_5 \\
ε
\end{array} →
\begin{array}{c}
q_6 \\
ε
\end{array} →
\begin{array}{c}
q_7 \\
a
\end{array} →
\begin{array}{c}
q_8 \\
b
\end{array} →
\begin{array}{c}
q_9 \\
ε
\end{array}
```

2. (a) Write down the definition of regular expressions over an alphabet Σ.

(b) Given a string \( w \), define \( w^R \) as the string \( w \) in reverse order. That is, if \( w = w_1w_2\ldots w_n \), then \( w^R = w_nw_{n-1}\ldots w_1 \). For example, if \( w = \text{live} \), then \( w^R = \text{evil} \).

Given a language \( L \), define its reversal \( L^R \) as the set of strings in \( L \) in reverse. More precisely, \( L^R = \{ w^R \mid w \in L \} \). For example, if \( L = \{ \text{live, raw, level} \} \), then \( L^R = \{ \text{evil, war, level} \} \).

If \( L \) is a regular language, prove that \( L^R \) is also regular.

3. Let \( L \) be any language. We say that two strings \( x \) and \( y \) are indistinguishable by \( L \) if for every string \( z \), we have \( xz \in L \) if and only if \( yz \in L \).

(a) For concreteness, consider \( L_1 = \{ x \in \{0, 1\}^* \mid \) the number of \( 1 \)'s in \( x \) is divisible by \( 3 \} \). Prove that \( 1 \) and \( 1111 \) are indistinguishable by \( L_1 \).

(b) Continuing with (a), which strings are indistinguishable from the string \( 1 \) by \( L_1 \)? The set of all such strings is the equivalence class of the string \( 1 \) and will be denoted by \([1]\).

(c) Find a string \( s \) not in \([1]\). What is the equivalence class of \( s \)? (We will denote this equivalence class by \([s]\))

(d) Can you find another string \( t \) not in \([1]\) or \([s]\)? What is the equivalence class of \( t \)?

(e) Can you find yet another string \( u \) not in these equivalence classes?

(f) Design a DFA for the language \( L_1 \). How are states in your DFA related to the equivalence classes?