Formal Languages and Automata Theory

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Chinese University of Hong Kong

Fall 2016
Welcome to 3130

www.cse.cuhk.edu.hk/~siuon/csci3130

Tentative syllabus and schedule

Textbook
Introduction to the Theory of Computation, Michael Sipser

Please sign up on piazza.com and ask questions
Or come to our office hours
AI everywhere

- Japanese cucumber farmer
  

- Georgia Tech’s Jill Watson
  
Is there anything that a computer cannot do?
Impossibilites

Why care about the impossible?

Example from Physics:
Since the Middle Ages, people tried to design machines that use no energy
Later physical discoveries forbid creating energy out of nothing
Perpetual motion is impossible

Understanding the impossible helps us to focus on the possible
Laws of computation

Just like laws of physics tell us what are (im)possible in nature...

$$\Delta U = Q + W$$

$$dS = \frac{\delta Q}{T}$$

$$S - S_0 = k_B \ln \Omega$$

Laws of computation tell us what are (im)possible to do with computers

Part of computer science

To some extent, laws of computation are studied in automata theory
Exploiting impossibilities

Certain tasks are believed impossible to solve quickly on current computers.

Given \( n = pq \) that is the product of two unknown primes, find \( p \) and \( q \).

Building block of cryptosystems.
Candy machine

Machine takes $5 and $10 coins
A gumball costs $15
Actions: +5, +10, Release
Slot machine

Why?
Different kinds of machines

Only one example of a machine

We will look at different kinds of machines and ask

- what kind of problems can this kind of machines solve?
- What are impossible for this kind of machines?
- Is machine \( A \) more powerful than machine \( B \)?
Some kinds of machines

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>finite automata</td>
<td>Devices with a small amount of memory</td>
</tr>
<tr>
<td></td>
<td>These are very simple machines</td>
</tr>
<tr>
<td>push-down automata</td>
<td>Devices with unbounded memory that can be accessed in a restricted way</td>
</tr>
<tr>
<td></td>
<td>Used to parse grammars</td>
</tr>
<tr>
<td>Turing machines</td>
<td>Devices with unbounded memory</td>
</tr>
<tr>
<td></td>
<td>These are actual computers</td>
</tr>
<tr>
<td>time-bounded Turing Machines</td>
<td>Devices with unbounded memory but bounded running time</td>
</tr>
<tr>
<td></td>
<td>These are computers that run fast</td>
</tr>
</tbody>
</table>
Course highlights

- Finite automata
  Closely related to pattern searching in text
  
  Find \((ab)^*(ab)\) in abracadabra

- Grammars
  - Grammars describe the meaning of sentences in English, and the meaning of programs in Java
  - Useful for natural language processing and compilers
Course highlights

Turing machines

- General model of computers, capturing anything we could ever hope to compute
- But there are many things that computers cannot do

Given the code of a program, tell if the program prints the string “3130”

```c
#include <stdio.h>
main(t,_,a)char *a; {return!0<t?t<3?main(-79,-13,a+main(-87,1_,
main(-86,0,a+1)+a)):
1:t?main(t+1,_,a):main(-94,-27+t,a)&&t==2?<_13?
main(2,_,1,%s %d %d\n")9:16:t<0?t<-72?mainn(_,_,
"%n",",*="/wq+\wcdnr+/,,[r/*de]+,/*[++,[/w(\r+/w#n+,/#{[l,+/n[n+,/+n+,/#
;#@n+,/*#:+,/+r :d"*3,}+[w+w'K:+'e#":dq="#l \n q#"+d'K#'!/+k#;q# "eKK#]w'r]eKK[nl]'/#;q#n'{)"#w'}}){nl}"+#/d;dw t;# 
}){nl}!n[#]; f{w'w r nc[nl]}/#{[l,+k {rw' iK[;[n]!/w#q'w\w kw nw' \ lww[KK[nl]!/w'w"w#" t; :{nl}"/[q#ld;r'][niw!/*/de]'c;\ 
 ;({nl'-}rw')++,#"#}*nc,*,#nw'/+kd'we)+;#'rdq#w! nr'/ ' })+{rl#'[n' 'k#
'}#{!}"}/
:t<-50?*a?putchar(31[a]):main(-65,_,a+1):main((*a='/')+t,_,a+1)
:0<t?main(2,2,"%s")!*a="/"||main(0,main(-61,*,a,
,"!ek;dc l@bK'(q)-[w]*n+r3#l,[]):nuwloca-O;m .vpbks,fxtndCeghry"),a+1);}
```

Does the program print “3130”?

Formal verification of software must fail on corner cases
Course highlights

Time-bounded Turing machines

- Many problems can be solved on a computer **in principle**, but takes too much time in practice

- **Traveling salesperson**: Given a list of cities, find the shortest way to visit them all and return home

  ![Diagram of cities]({seoul}, {hong_kong}, {shanghai}, {tokyo}, {bangkok}, {manila}, {taipei})

- For 100 cities, takes **100+ years** to solve even on the fastest computer!
Problems we will look at

Can machine $A$ solve problem $B$?

- Examples of problems we will consider
  - Given a word $s$, does it contain “to” as a subword?
  - Given a number $n$, is it divisible by 7?
  - Given two words $s$ and $t$, are they the same?

- All of these have “yes/no” answers (decision problems)

- There are other types of problems, like “Find this” or “How many of that” but we won’t look at them
Strings are a common way to talk about words, numbers, pairs of numbers.

Which symbols can appear in a string? As specified by an alphabet.

An alphabet is a finite set of symbols.

Examples:

$\Sigma_1 = \{a, b, c, d, \ldots, z\}$: the set of English letters

$\Sigma_2 = \{0, 1, 2, \ldots, 9\}$: the set of digits (base 10)

$\Sigma_3 = \{a, b, c, \ldots, z, \#\}$: the set of letters plus the special symbol #
Strings

An input to a problem can be represented as a string

A string over alphabet $\Sigma$ is a finite sequence of symbols in $\Sigma$

- axyzzy is a string over $\Sigma_1 = \{a, b, c, \ldots, z\}$
- 3130 is a string over $\Sigma_2 = \{0, 1, \ldots, 9\}$
- ab#bc is a string over $\Sigma_3 = \{a, b, \ldots, z, \#\}$

- The empty string will be denoted by $\varepsilon$
  (What you get using """" in C, Java, Python)

- $\Sigma^*$ denotes the set of all strings over $\Sigma$
  All possible inputs using symbols from $\Sigma$ only
A language is a set of strings (over the same alphabet)

Languages describe problems with “yes/no” answers:

\[ L_1 = \text{All strings containing the substring “to”} \quad \Sigma_1 = \{a, \ldots, z\} \]

stop, to, toe are in \( L_1 \)
\( \varepsilon, \) oyster are not in \( L_1 \)

\[ L_1 = \{x \in \Sigma_1^* \mid x \text{ contains the substring “to”}\} \]
Examples of languages

\[ L_2 = \{ x \in \Sigma_2^* \mid x \text{ is divisible by 7} \} \quad \Sigma_2 = \{0, 1, \ldots, 9\} \]

\[ L_2 \text{ contains 7, 14, 21, \ldots} \]
Examples of languages

\[ L_2 = \{ x \in \Sigma_2^* \mid x \text{ is divisible by } 7 \} \quad \Sigma_2 = \{0, 1, \ldots, 9\} \]

\( L_2 \) contains 7, 14, 21, …

\[ L_3 = \{ s#s \mid s \in \{a, \ldots, z\}^* \} \quad \Sigma_3 = \{a, b, \ldots, z, \#\} \]

Which of the following are in \( L_3 \)?

- ab#ab
- ab#ba
- a##a#
Examples of languages

\[ L_2 = \{ x \in \Sigma_2^* \mid x \text{ is divisible by 7} \} \quad \Sigma_2 = \{0, 1, \ldots, 9\} \]

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\[ L_3 = \{ s#s \mid s \in \{a, \ldots, z\}^* \} \quad \Sigma_3 = \{a, b, \ldots, z, \#\} \]

Which of the following are in \( L_3 \)?

<table>
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<tr>
<th>String</th>
<th>Answer</th>
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<tbody>
<tr>
<td>ab#ab</td>
<td>Yes</td>
</tr>
<tr>
<td>ab#ba</td>
<td>No</td>
</tr>
<tr>
<td>a##a#</td>
<td>No</td>
</tr>
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</table>
Finite Automata
Example of a finite automaton

- There are states $0$, $5$, $10$, go
- The start state is $0$
- Takes inputs from $\{+5, +10, R\}$
- The state go is an accepting state
- There are transitions specifying where to go to for every state and every input symbol
A finite automaton (DFA) is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\) where

- \(Q\) is a finite set of states
- \(\Sigma\) is an alphabet
- \(\delta : Q \times \Sigma \rightarrow Q\) is a transition function
- \(q_0 \in Q\) is the initial state
- \(F \subseteq Q\) is the set of accepting states (or final states)

In diagrams, the accepting states will be denoted by double circles.
Example

alphabet $\Sigma = \{0, 1\}$
states $Q = \{q_0, q_1, q_2\}$
initial state $q_0$
accepting states $F = \{q_0, q_1\}$

<table>
<thead>
<tr>
<th>states</th>
<th>inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>$q_0$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$q_2$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$q_2$</td>
</tr>
</tbody>
</table>
A DFA accepts a string $x$ if starting from the initial state and following the transition as $x$ is read from left to right, the DFA ends at an accepting state.

The DFA accepts 0 and 011 but not 10 and 0101.

The language of a DFA is the set of all strings $x$ accepted by the DFA.

0 and 011 are in the language. 10 and 0101 are not.
The languages of these DFAs?

\[ \Sigma = \{a, b\} \]

![Diagram of a DFA with states \( q_0 \) and \( q_1 \). Transitions are labeled 'a' and 'b'.]

\[ \Sigma = \{a, b\} \]

![Diagram of a DFA with states \( q_0 \) and \( q_3 \), \( q_2 \) and \( q_4 \). Transitions are labeled 'a' and 'b'.]

\[ \Sigma = \{0, 1\} \]

![Diagram of a DFA with states \( q_0 \), \( q_1 \), and \( q_2 \). Transitions are labeled '0', '1', and '0,1'.]
Examples

Construct a DFA over alphabet \{0, 1\} that accepts all strings with at most three 1s
Examples

Construct a DFA over alphabet \( \{0, 1\} \) that accepts all strings with at most three 1s

\[
\begin{array}{c}
q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_4+ \\
q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_4+ \\
q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_4+ \\
q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_4+ \\
q_0 \xrightarrow{0} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{1} q_3 \xrightarrow{1} q_4+
\end{array}
\]
Examples

Construct a DFA over alphabet \( \{0, 1\} \) that accepts all strings ending in 01.
Examples

Construct a DFA over alphabet \(\{0, 1\}\) that accepts all strings ending in 01.

Hint: The DFA must "remember" the last 2 bits of the input string.
Construct a DFA over alphabet \( \{0, 1\} \) that accepts all strings ending in 01.

Hint: The DFA must “remember” the last 2 bits of the input string.
Examples

Construct a DFA over alphabet \( \{0, 1\} \) that accepts all strings ending in 101.