Week 5 Tutorial Session

1. For any integer $k \geq 0$, define $L_k = \{ww \mid w \in \{0, 1\}^k\}$.
   
   (a) Write down all strings in $L_3$.
   
   (b) Prove that any DFA for $L_k$ has at least $2^k$ states.

2. Let $L$ be the set of strings over $\{0, 1\}$ whose number of ones is a perfect square (e.g. 0, 1, 4, 9, 16, ...).
   Prove that $L$ is irregular.

3. This problem concerns the following DFA.

   (a) Run the minimization algorithm on this DFA. Show the table of pairs of distinguishable states at the end of the algorithm. Also draw the minimized DFA.

   (b) Show that every pair of states in the minimized DFA is distinguishable.

   (c) Convert the minimized DFA into a regular expression using the conversion algorithm from class. Show the preprocessing step and how the NFA changes after each state is eliminated.